Expanding the Universe –
From Volume Rendering to
High-Dimensional Data Visualization

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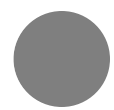
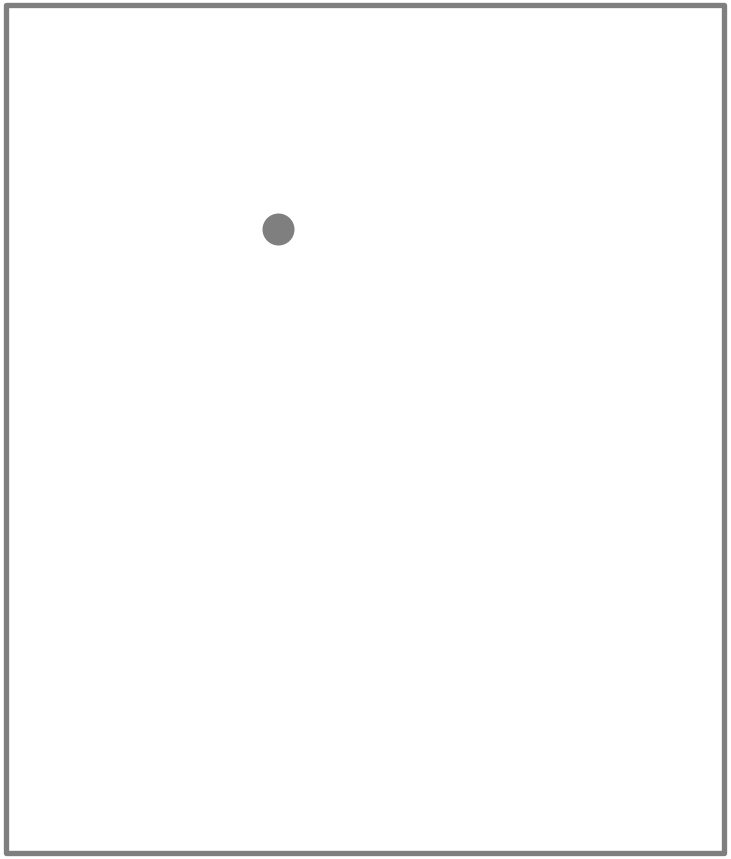
A Point

A Point

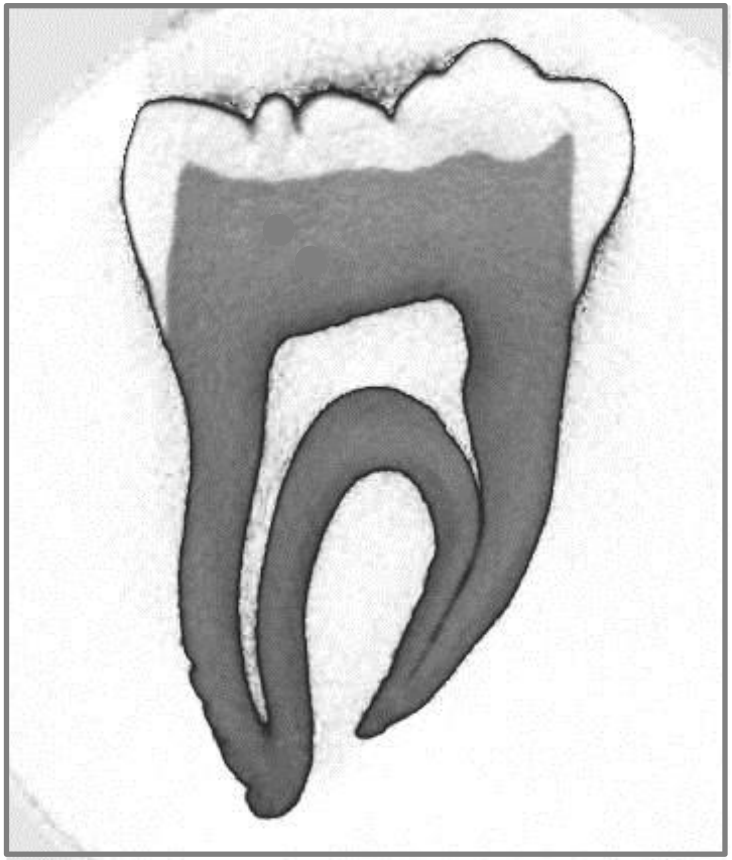


A Point

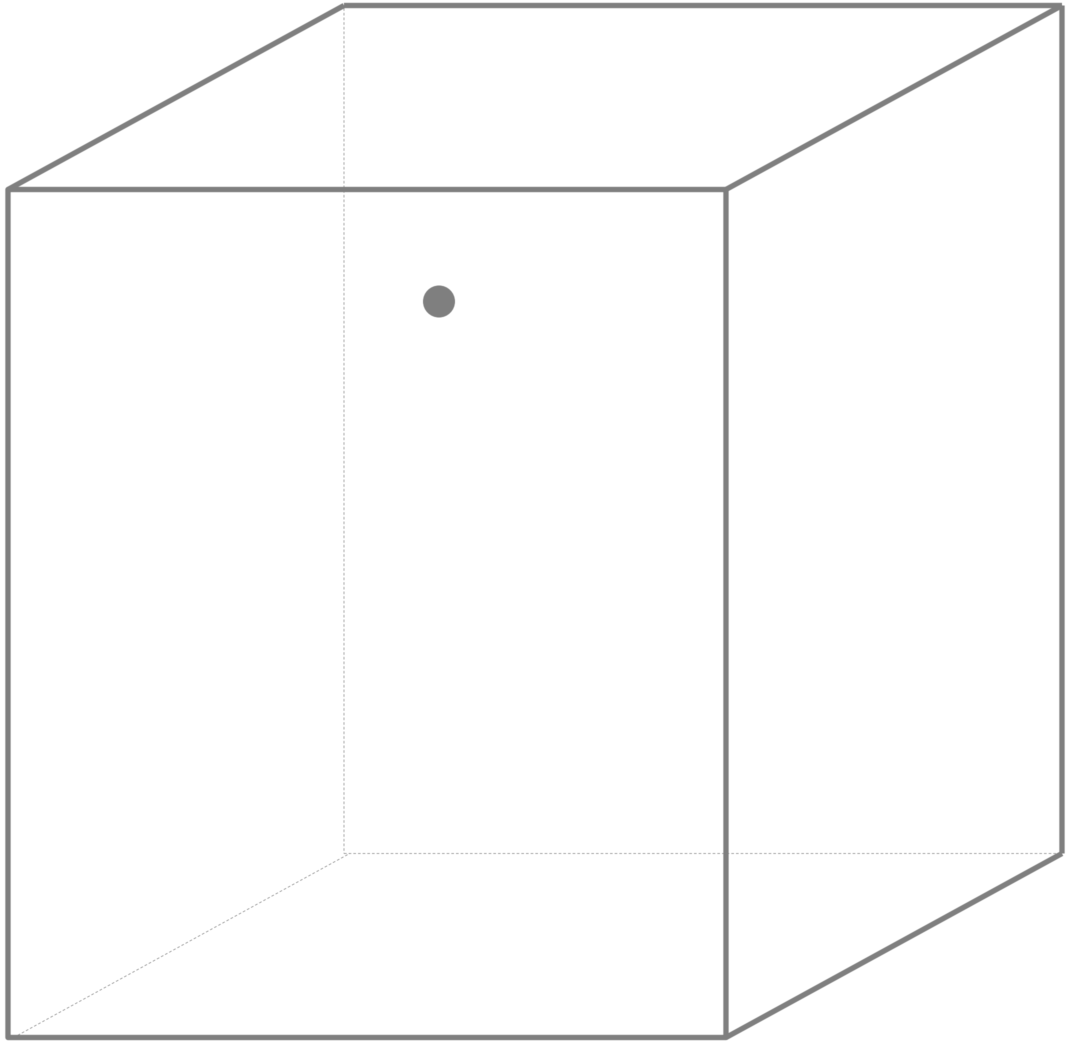
A Point in a Frame



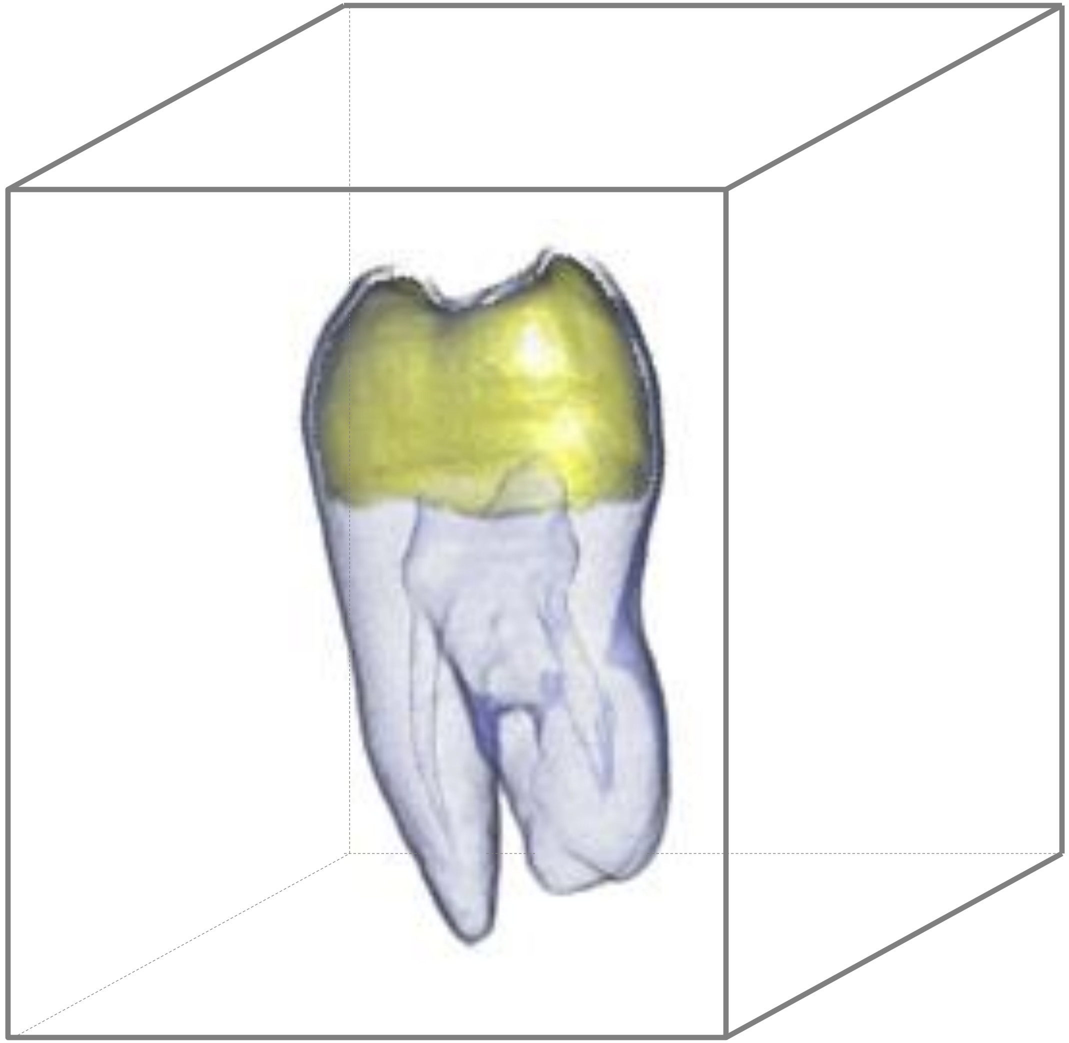
Many Points in a Frame → An Image



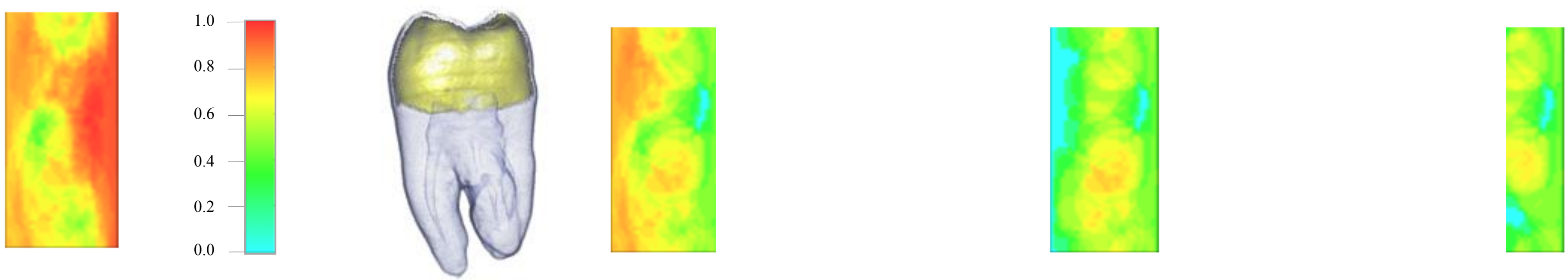
A Point in a Cube



Many Points in a Cube → A Volumetric Image



Navigating the Volume



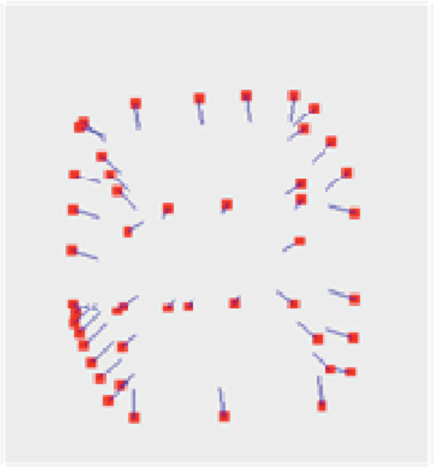
View Suggestion Framework

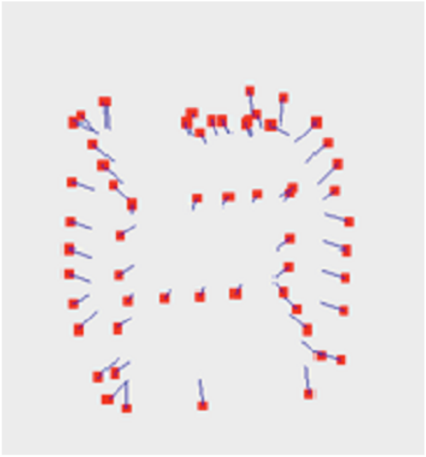
Features and Clustering

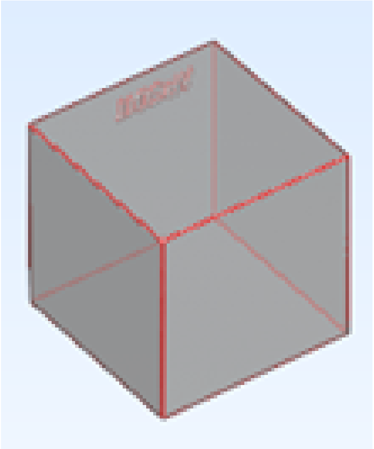
Feature – normal perturbation within a small neighborhood

$$w(x_0, y_0, z_0) = \sum_{(x,y,z)\in N(x_0,y_0,z_0)} |\nabla f(x,y,z) - \nabla f(x_0,y_0,z_0)|$$

Example – cube with and without text

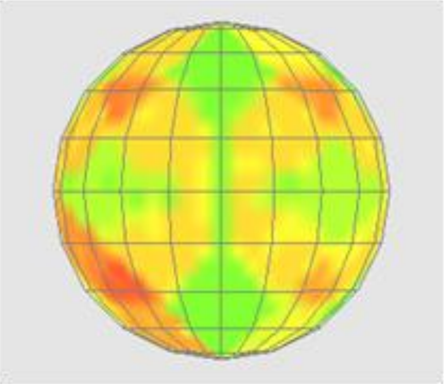
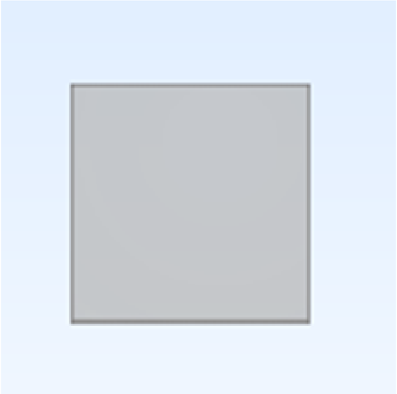
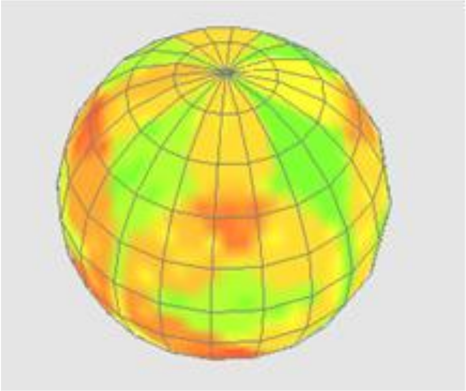
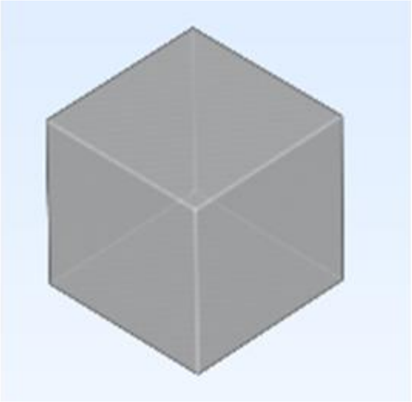






Interaction with Embedded Navigation Aids

Trackball with entropy map

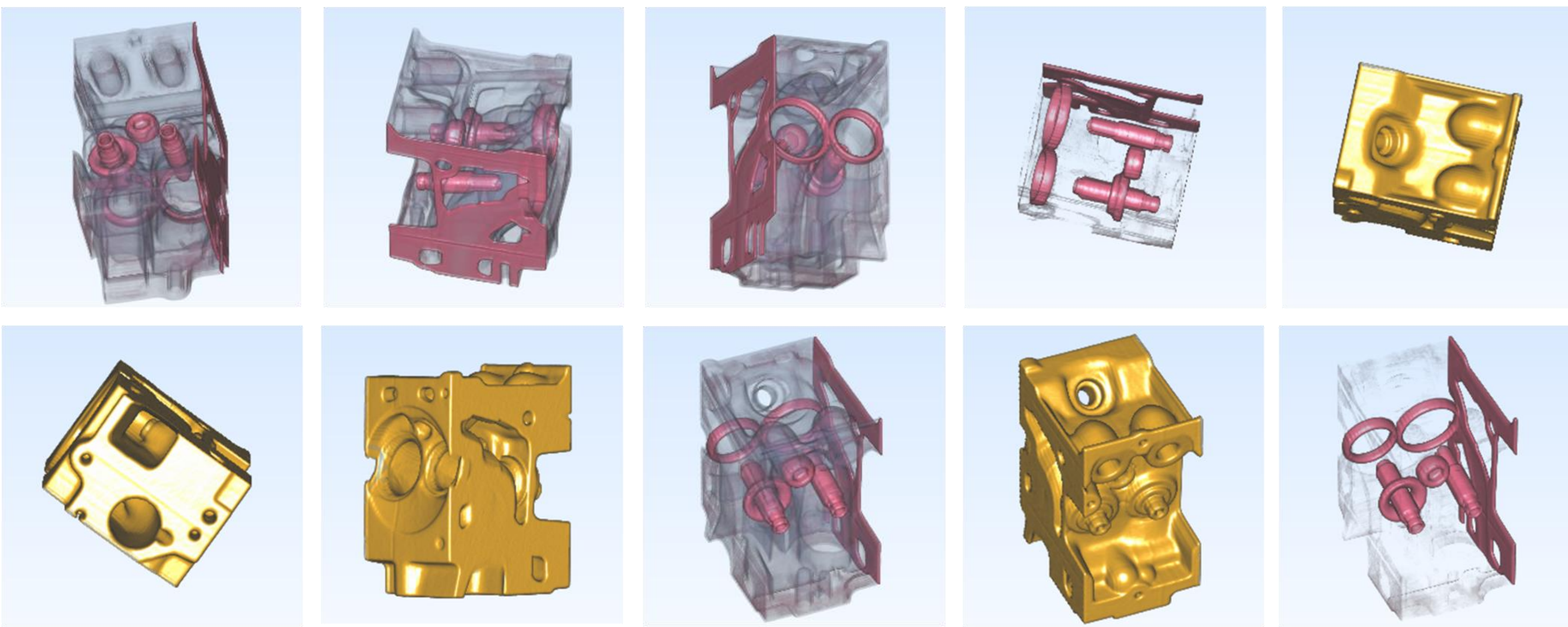
Video

Z. Zheng, N. Ahmed, K. Mueller, “iView: Feature Clustering Framework for Suggesting Informative Views in Volume Visualization,” IEEE Trans. on Visualization and Computer Graphics, 17(12):1959-1968, 2011 (video)

presented at VIS 2011

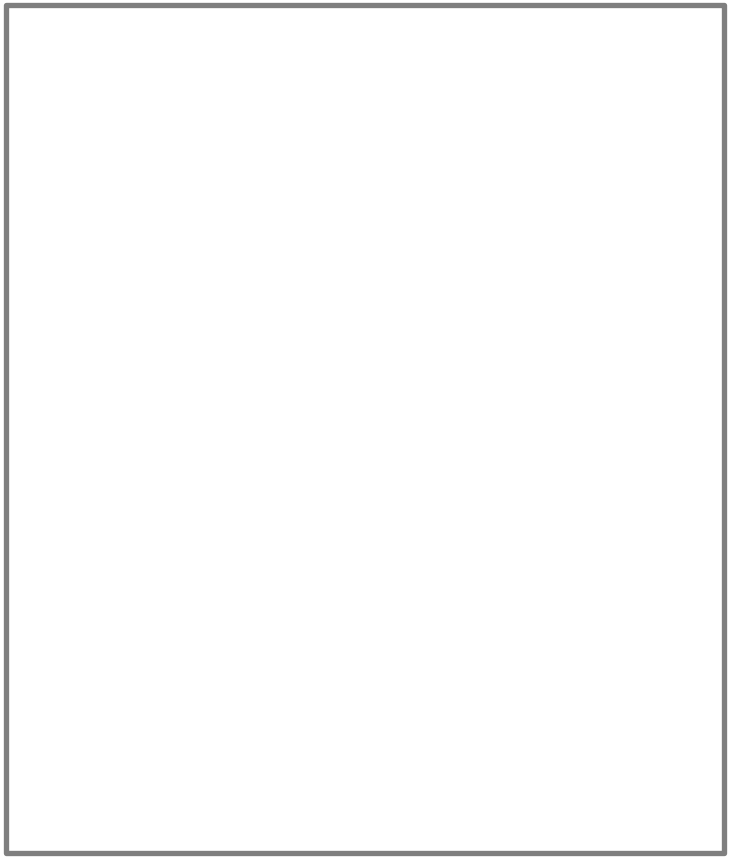
Transfer Function Independence

May still need multiple transfer functions to see all features

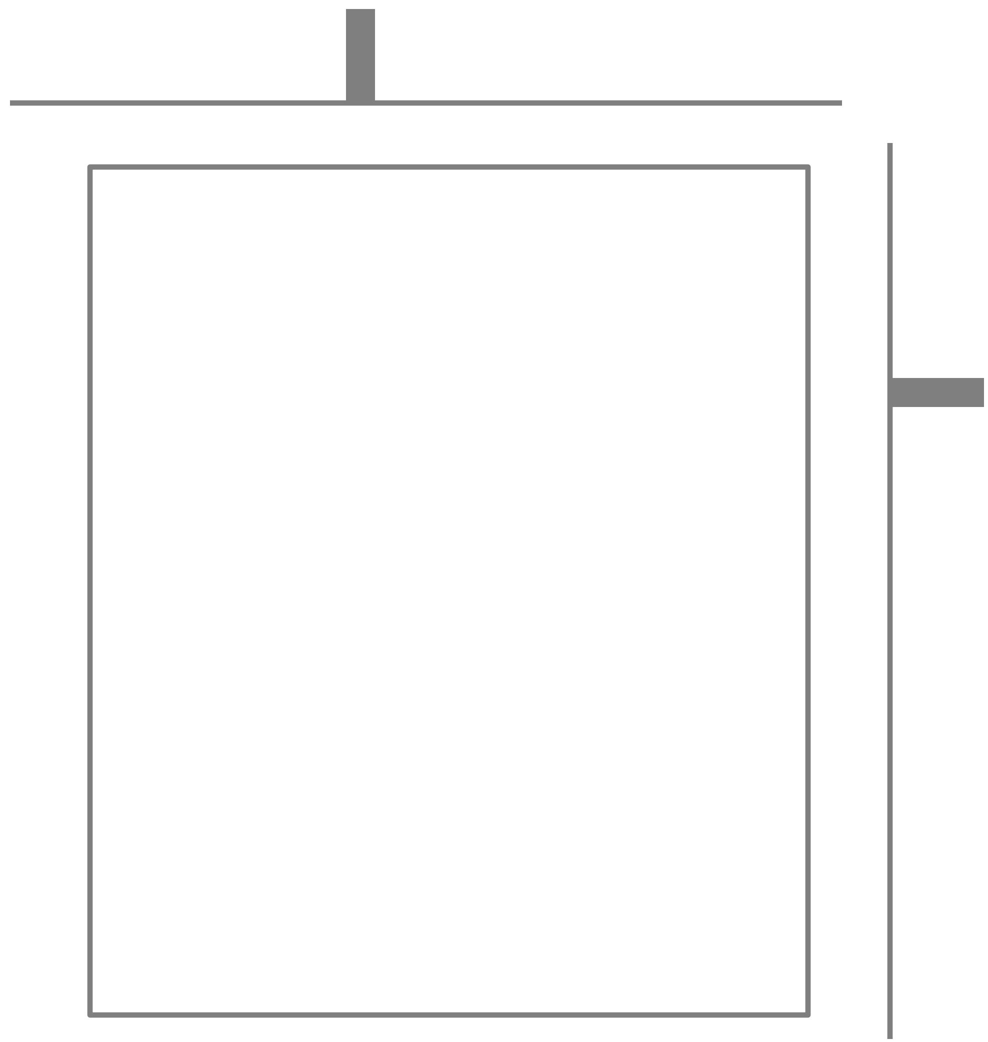


Volume Generation

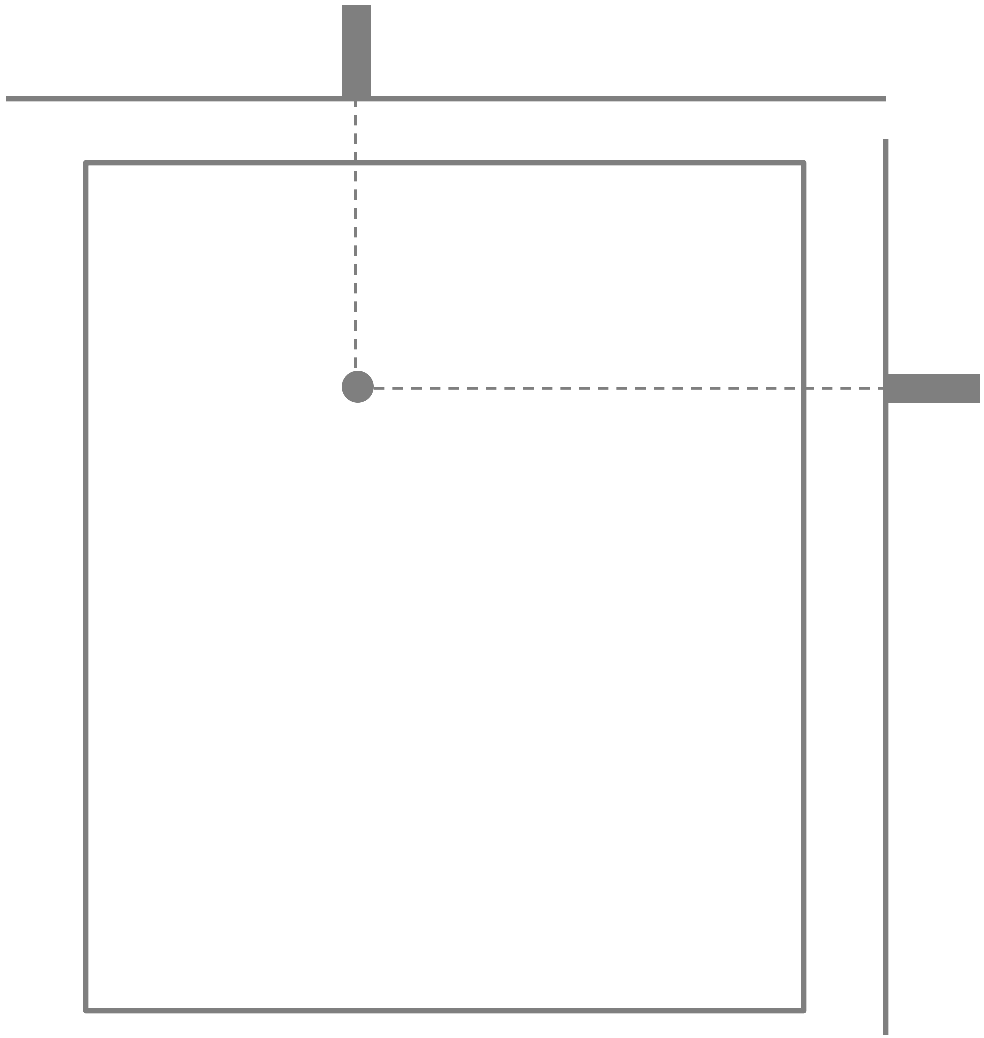
A Frame



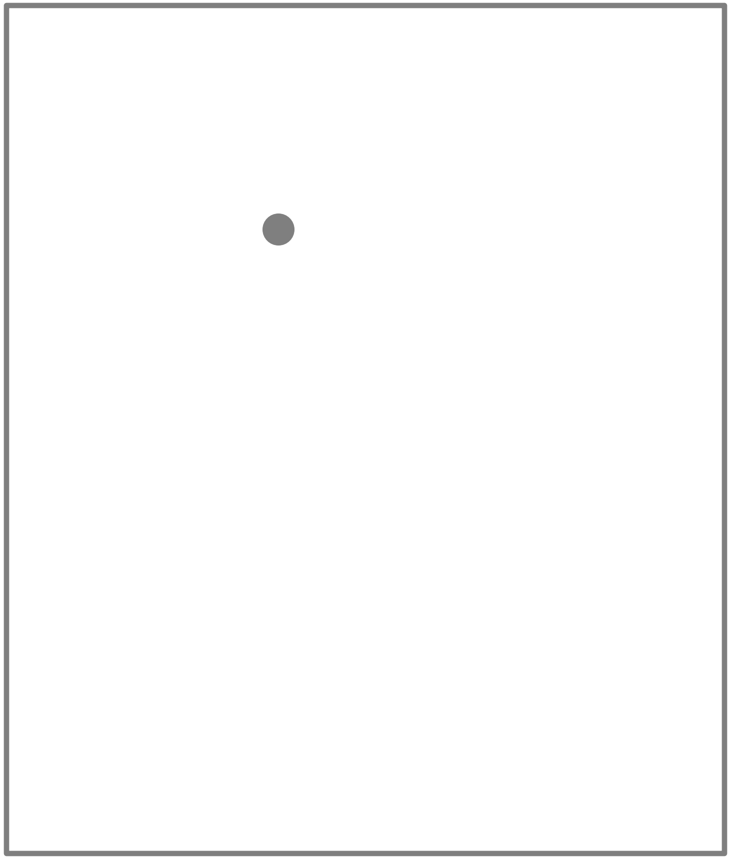
A Frame and Views



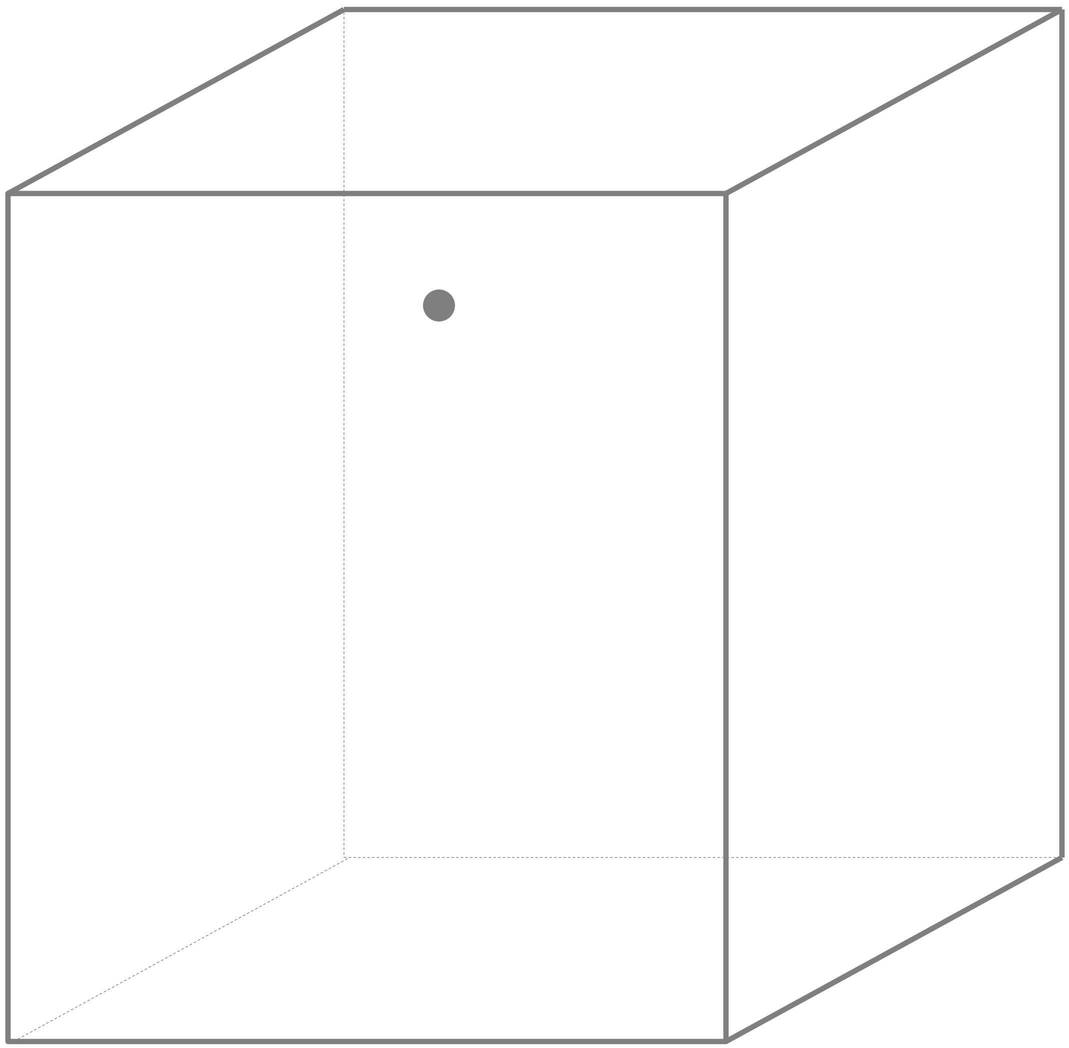
A Point Reconstructed from the Views



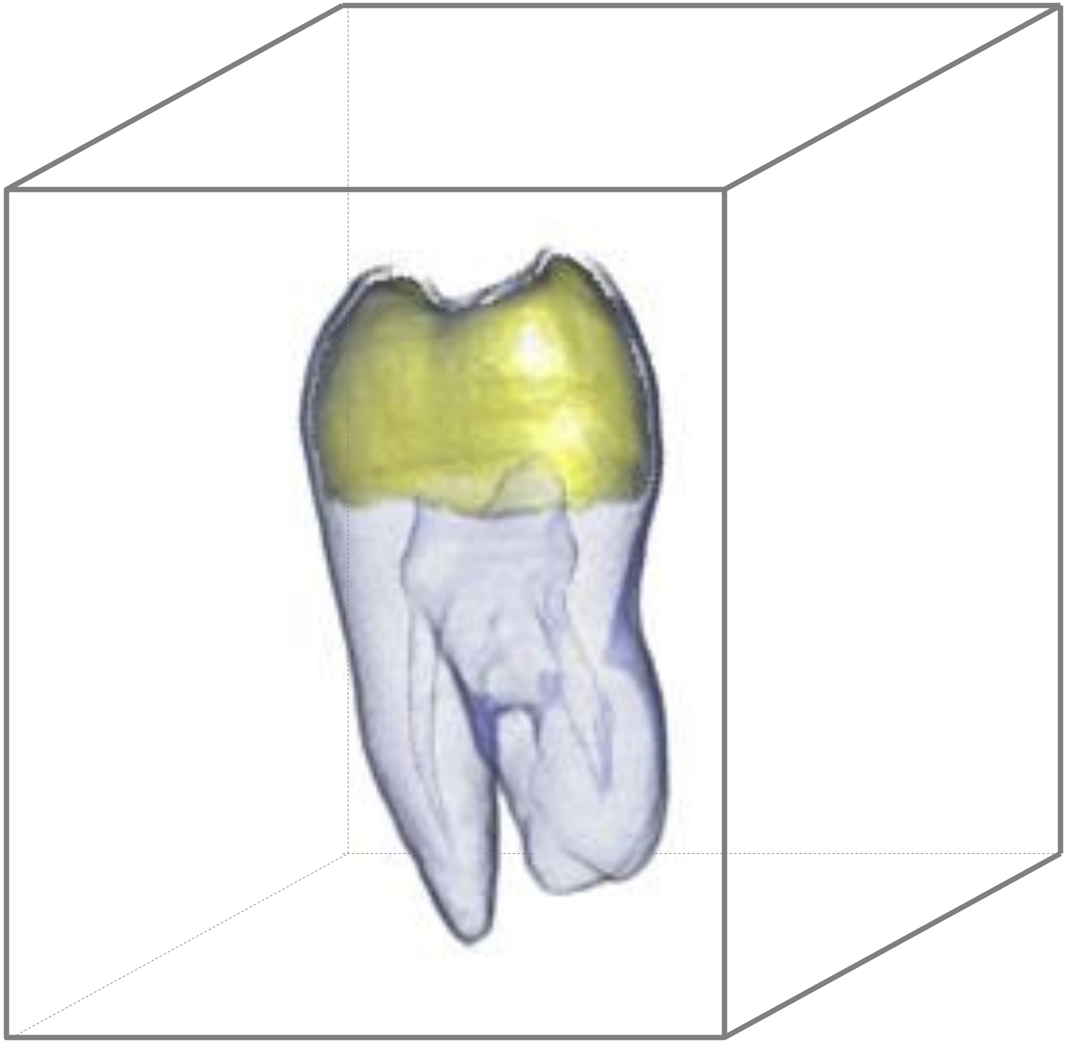
A Point in a Frame



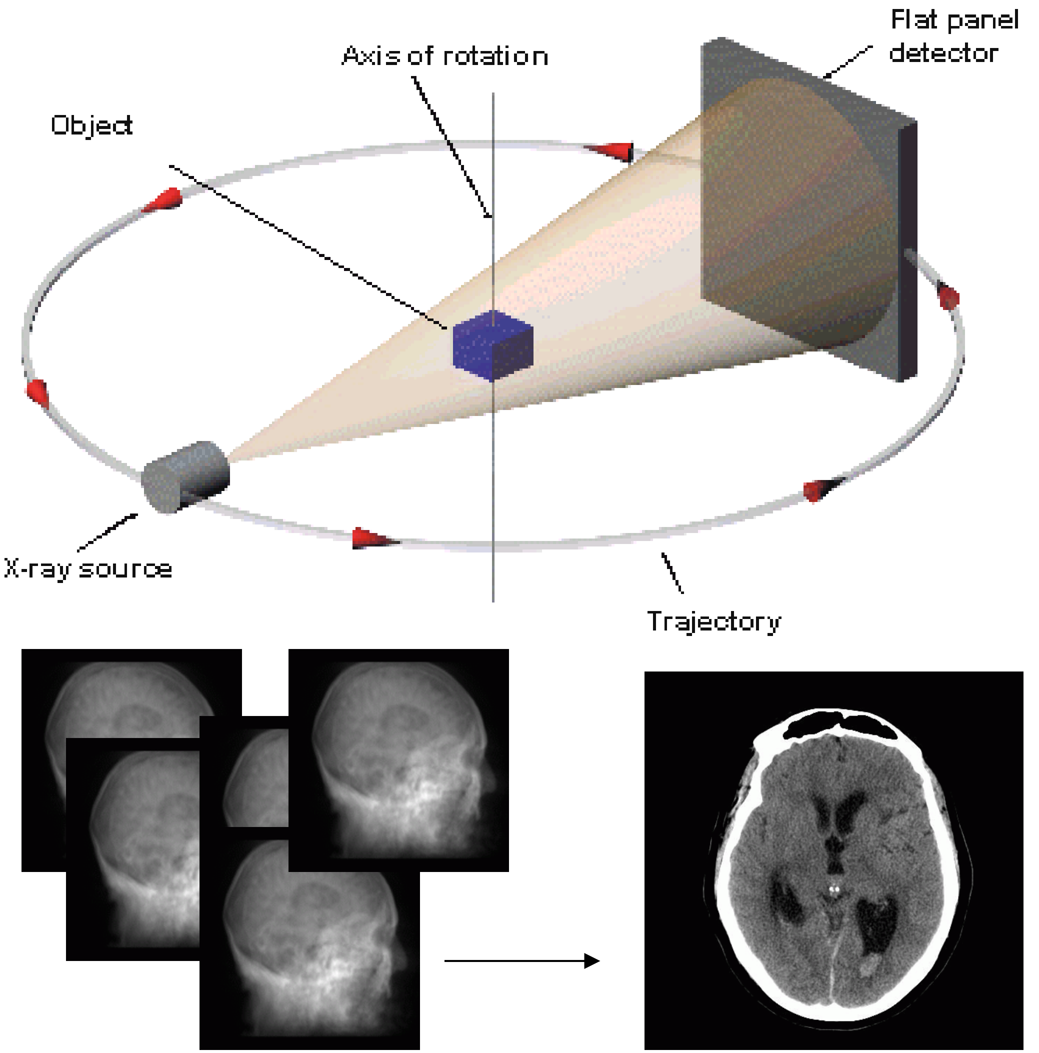
A Point in a Cube



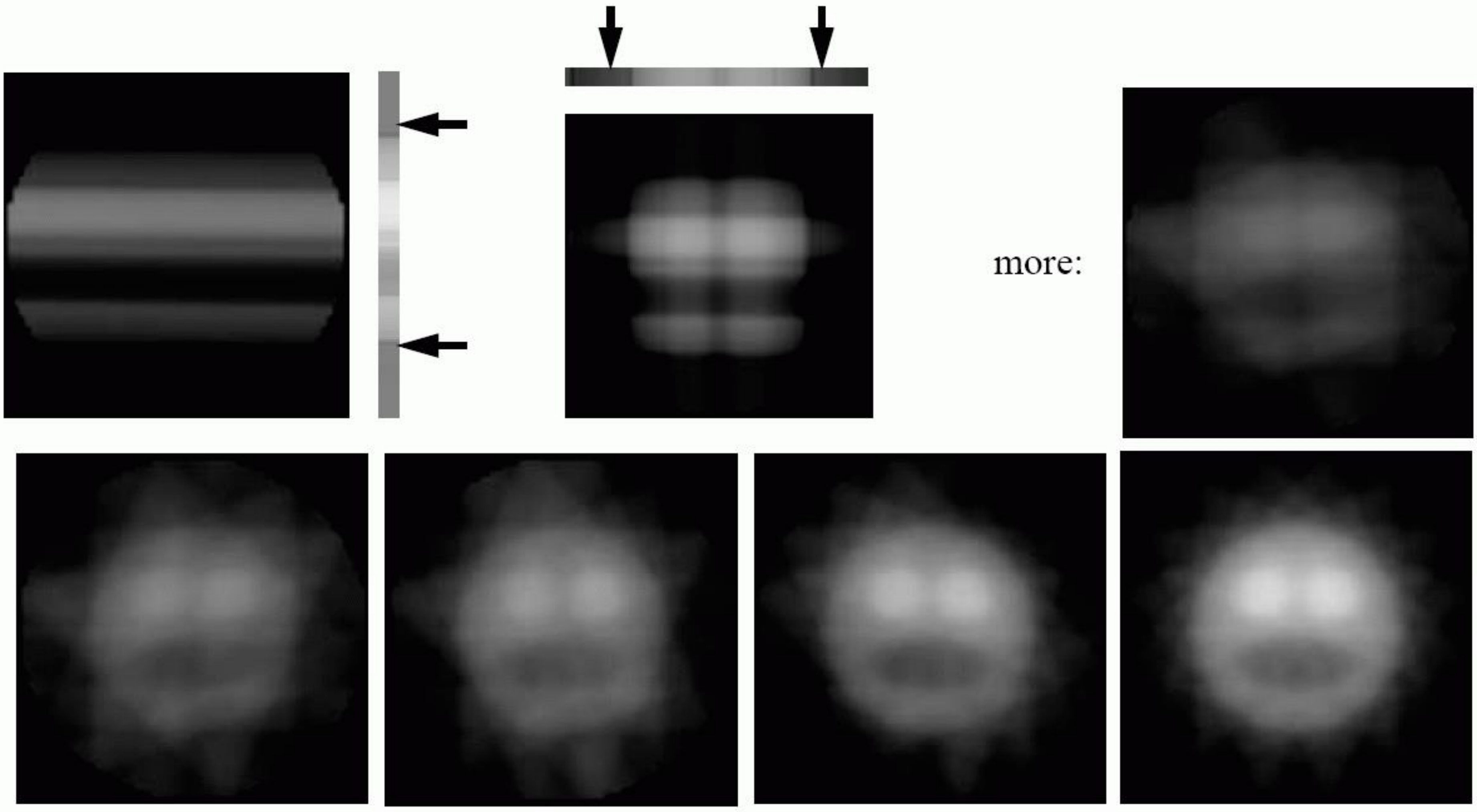
Many Points in a Cube



Computed Tomography

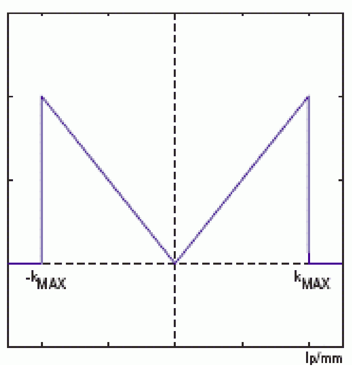


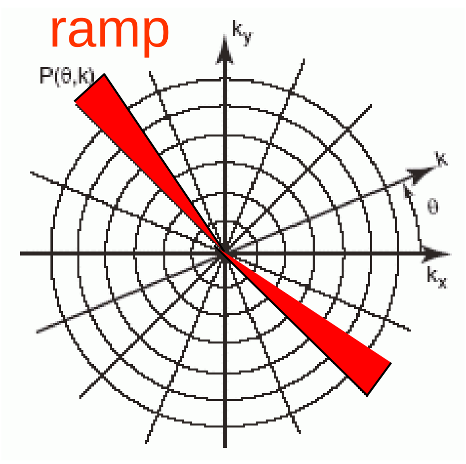
Backprojection



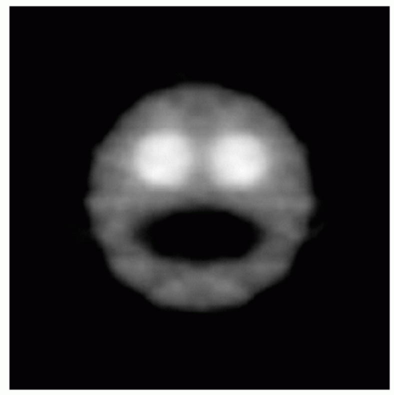
Filtered Backprojection

First filter projection in the frequency domain





Then backproject

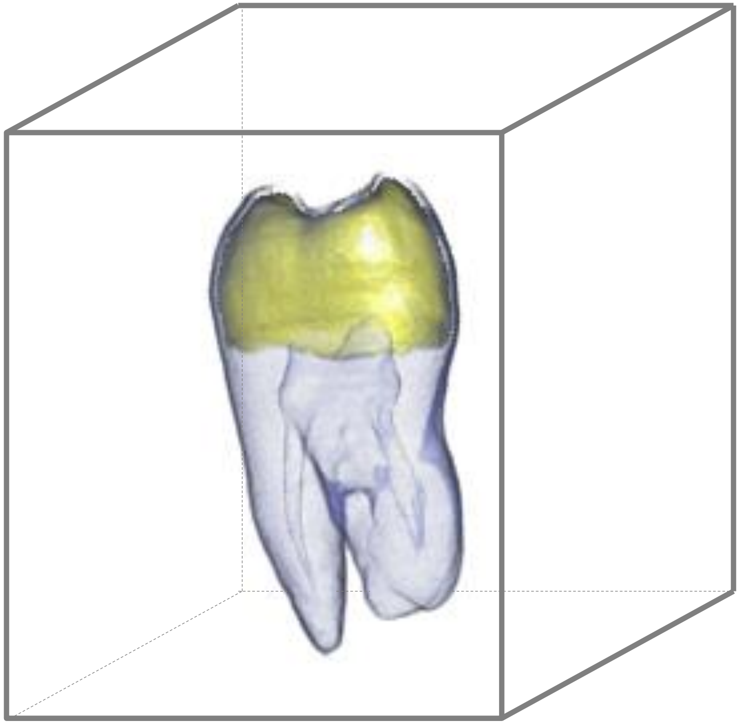


3D

Comprehension ✓

Navigation ✓

Reconstruction ✓



>> 3D

Consider the salient features of a car:

- miles per gallon (MPG)
- top speed
- acceleration
- number of cylinders
- horsepower
- weight
- year
- country origin
- brand
- number of seats
- number of doors
- reliability (average number of breakdowns)
- and so on...

What Does This Object Look Like?

A1 fx Urban population

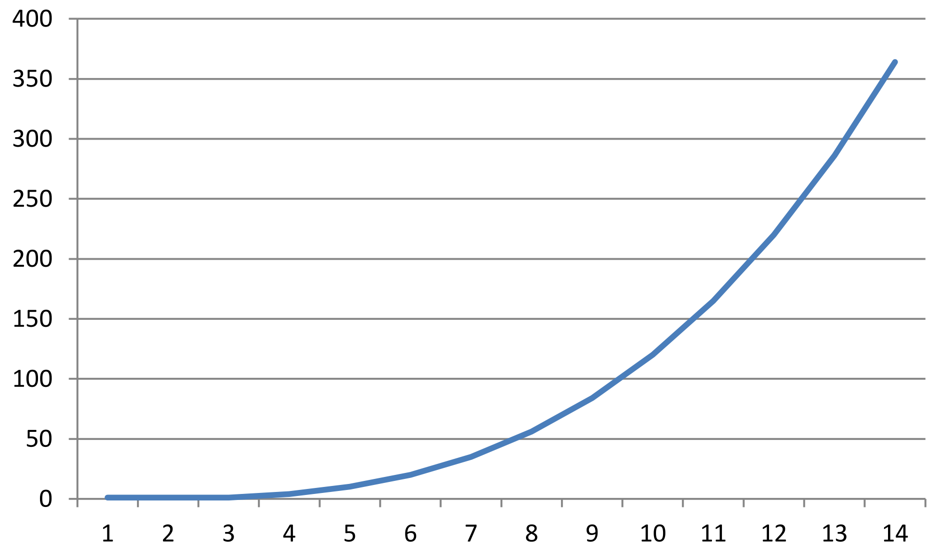
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Urban population	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974
2	Afghanistan	769308	811389	855131	900646	948060	997499	1053104	1110728	1170961	1234664	1302370	1391081	1483942	1579748	1676656
3	Albania	494443	511637	529182	547024	565117	583422	601897	620508	639234	658062	676985	698179	719561	741149	762972
4	Algeria	3293999	3513320	3737362	3969886	4216744	4483048	4644898	4822860	5015071	5218184	5429743	5618190	5813978	6017932	6231383
5	American Samoa	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	Andorra	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	Angola	521205	552777	585121	618345	652638	688181	729595	772643	817418	863993	912486	982944	1056617	1133936	1215437
8	Antigua and Barbuda	21699	21737	21878	22086	22309	22513	22717	22893	23053	23218	23394	24046	24718	25342	25826
9	Argentina	15224096	15588864	15957125	16328045	16700303	17073371	17432905	17793789	18160868	18540720	18938137	19335571	19750609	20180707	20621674
10	Armenia	957974	1008899	1061551	1115546	1170414	1225785	1281346	1337060	1393199	1450241	1508526	1565054	1622558	1680709	1739019
11	Aruba	24996	25514	26019	26498	26941	27337	27683	27984	28247	28491	28726	28959	29188	29409	29610
12	Australia	8375329	8585577	8840666	9055650	9279777	9508980	9770529	9937118	10157212	10416192	10668471	11050785	11271606	11461308	11771589
13	Austria	4560057	4589541	4621666	4653194	4685421	4715750	4754585	4778506	4798552	4817322	4849178	4871380	4904030	4932109	4939292
14	Azerbaijan	1857673	1929429	2004258	2080816	2157307	2232355	2306310	2378380	2448728	2517815	2586000	2660687	2734631	2807879	2880491
15	Bahamas	65457	69655	74179	78961	83902	88918	93931	98974	103944	108721	113219	117339	121142	124761	128393
16	Bahrain	128480	133815	139791	146052	152097	157596	162844	167630	172373	177677	183997	191379	199768	209201	219678
17	Bangladesh	2761049	2947191	3141372	3344120	3556037	3777716	4047121	4329144	4624445	4933701	5257558	5710277	6184871	6682073	7202503
18	Barbados	84884	85284	85761	86285	86797	87259	87707	88117	88526	88986	89532	90518	91596	92713	93796
19	Belarus	2656152	2774166	2896449	3022217	3150553	3280410	3415984	3554673	3695363	3836802	3977600	4131179	4285735	4439788	4591705
20	Belgium	8435075	8489549	8548773	8620194	8709437	8796088	8865259	8924327	8968568	9003536	9040444	9086816	9134227	9175144	9217085
21	Belize	49165	50608	52156	53734	55226	56561	57756	58820	59746	60532	61186	61883	62445	62984	63665
22	Benin	211033	229172	248065	267765	288321	309788	337282	366019	396065	427482	460341	500355	542251	586179	632320
23	Bermuda	44400	45500	46600	47700	48900	50100	51000	52000	53000	54000	55000	54600	54200	53800	53400
24	Bhutan	8064	8778	9526	10311	11137	12010	13089	14230	15445	16750	18158	19926	21827	23858	26008
25	Bolivia	1233398	1271250	1310294	1350615	1392328	1435536	1480255	1526529	1574517	1624419	1676370	1730434	1786553	1844596	1904355
26	Bosnia and Herzegovi	604204	637337	671124	705395	739884	774380	812856	851325	890011	929301	969514	1008688	1048890	1089898	1131315
27	Botswana	16240	17379	18583	19855	21203	22631	28191	34090	40352	46995	54038	61638	69689	78254	87422
28	Brazil	32662018	34463344	36353068	38320171	40346703	42418482	44548227	46722996	48945984	51223962	53563179	56042505	58587770	61207586	63913385
29	Brunei	35501	38753	42173	45802	49699	53916	58461	63355	68595	74157	80024	83802	87671	91616	95629
30	Bulgaria	2918659	3085061	3251675	3418610	3588246	3756058	3889518	4022040	4159890	4301340	4440270	4554810	4667059	4782931	4907107
31	Burkina Faso	221872	230199	238713	247472	256558	266039	275958	286311	297074	308196	319642	332556	345877	359655	373966
32	Burundi	58810	61055	63344	65696	68137	70683	73370	76186	79034	81779	84324	90879	97308	103757	110494
33	Cambodia	559631	578678	598248	618631	640243	663272	747219	835638	927177	1019449	1110079	962037	806676	645287	479631
34	Cameroon	751711	801009	852578	906523	962928	1021891	1088521	1158289	1231375	1307967	1388275	1522958	1664410	1813278	1970385
35	Canada	12375125	12764121	13145207	13536503	13941055	14345262	14727261	15108962	15470875	15800439	16142268	16381341	16640381	16920220	17221765
36	Cape Verde	32791	34353	35972	37672	39487	41435	43592	45884	48200	50383	52314	54103	55620	56940	58184
37	Cayman Islands	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
38	Central African Rep.	302157	317715	333986	351001	368787	387357	408129	429825	452326	475441	499036	526414	554452	583376	613530
39	Chad	198777	213406	228652	244499	260903	277834	305390	333898	363523	394530	427153	467662	510348	554973	601045
40	Channel Islands	42565	42665	42792	42941	43102	43269	43437	43604	43765	43916	44051	44028	43987	43907	43762

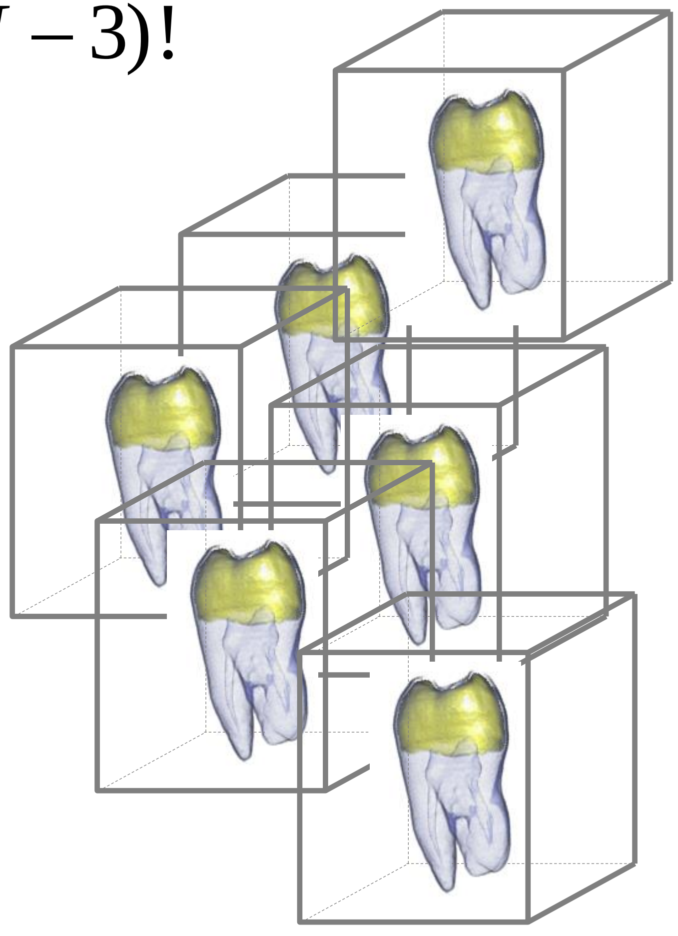
Hard to imagine....

How Many 3D Cubes in N-D?

Combinatorial explosion

$$\frac{N!}{3!(N-3)!}$$

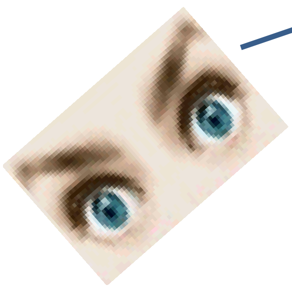


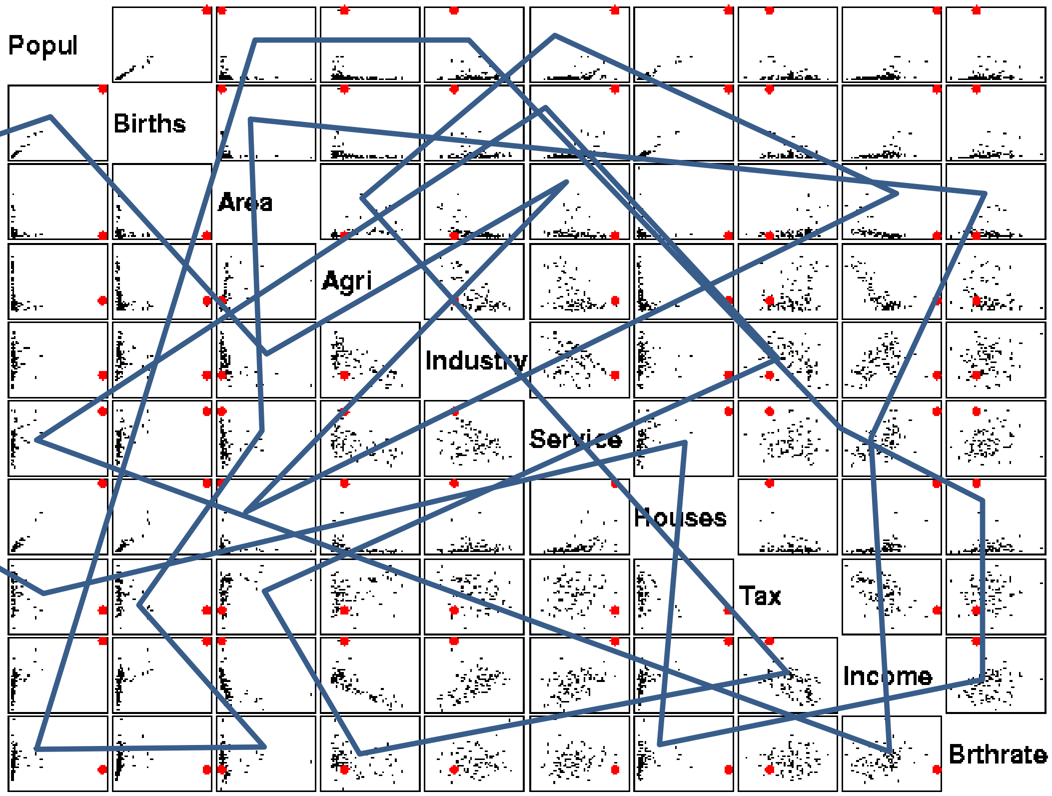


Scatterplot Matrices Don't Scale

Can't see multivariate relationships

- especially not when D is high





What You Need Is…

tripadvisor ND

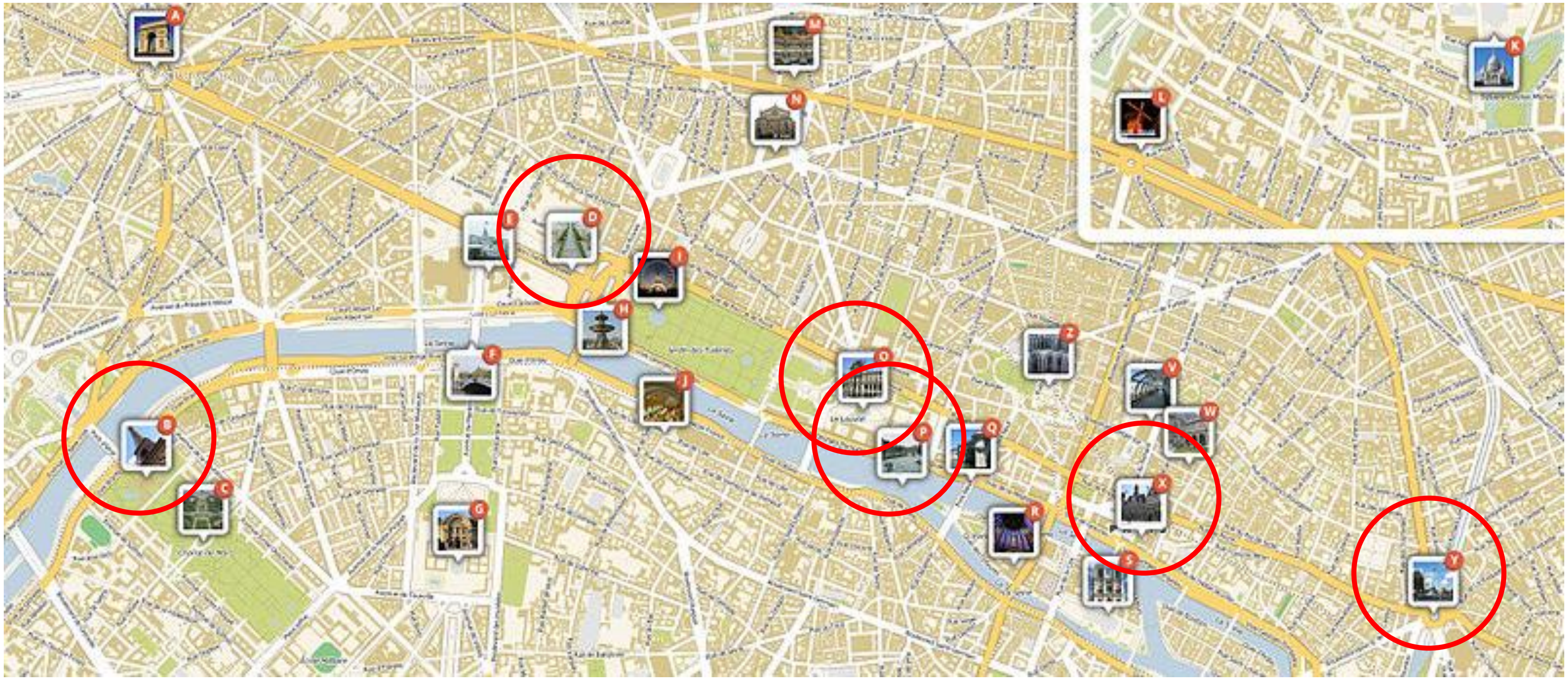
Taking a Trip in Real Life

1. Identify the sights

- use a map to identify the sights of interest and their location

Taking a Trip in Real Life

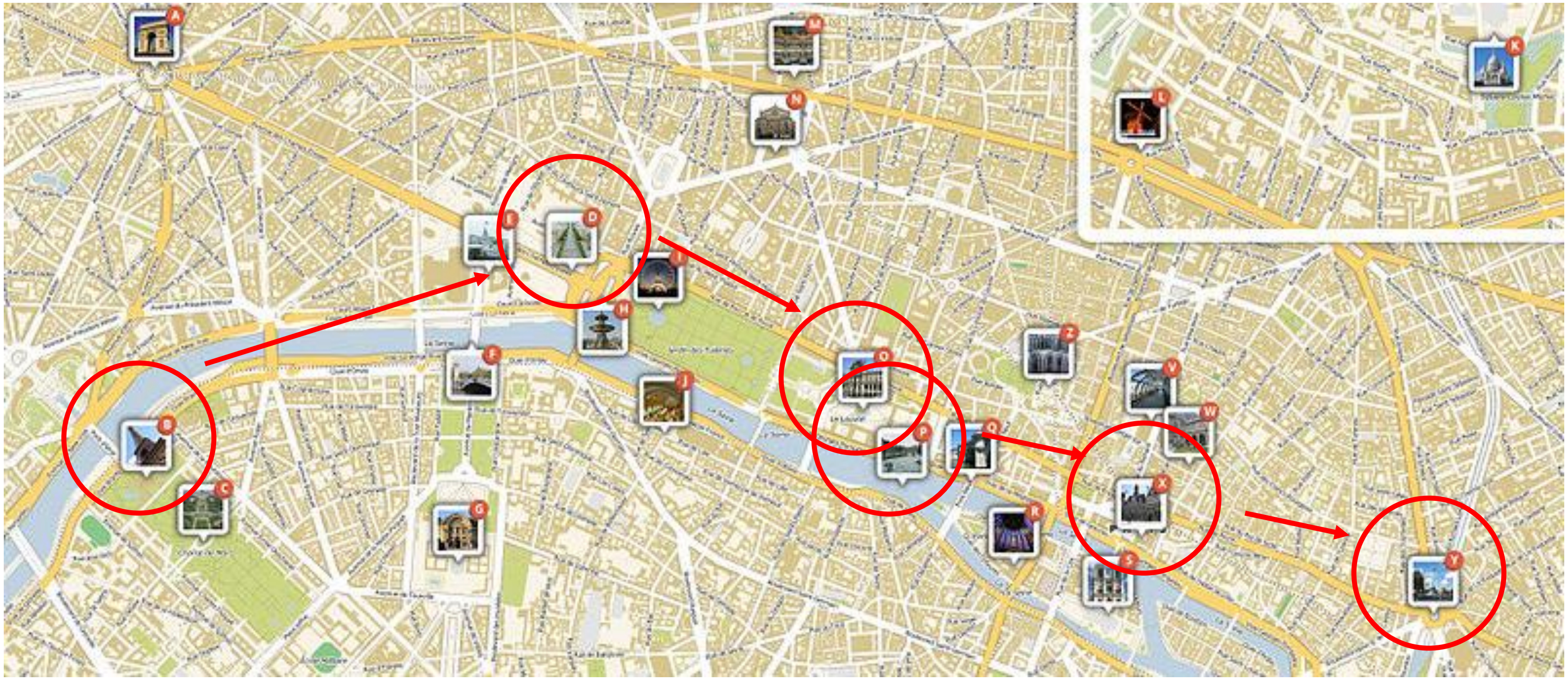
1. Identify the sights
   - use a map to identify the sights of interest and their location



Taking a Trip in Real Life

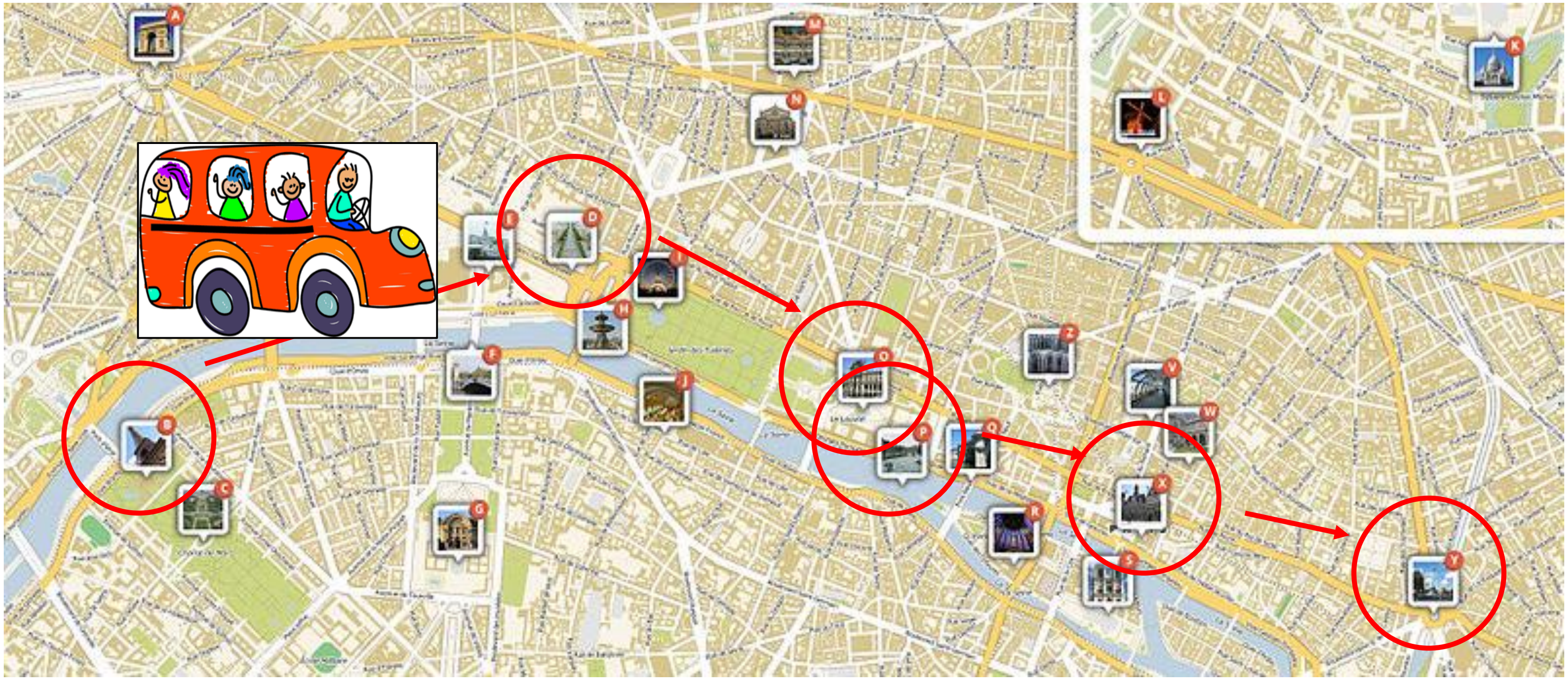
2. Plan the trip

- connect the sights of interest along a path



Taking a Trip in Real Life

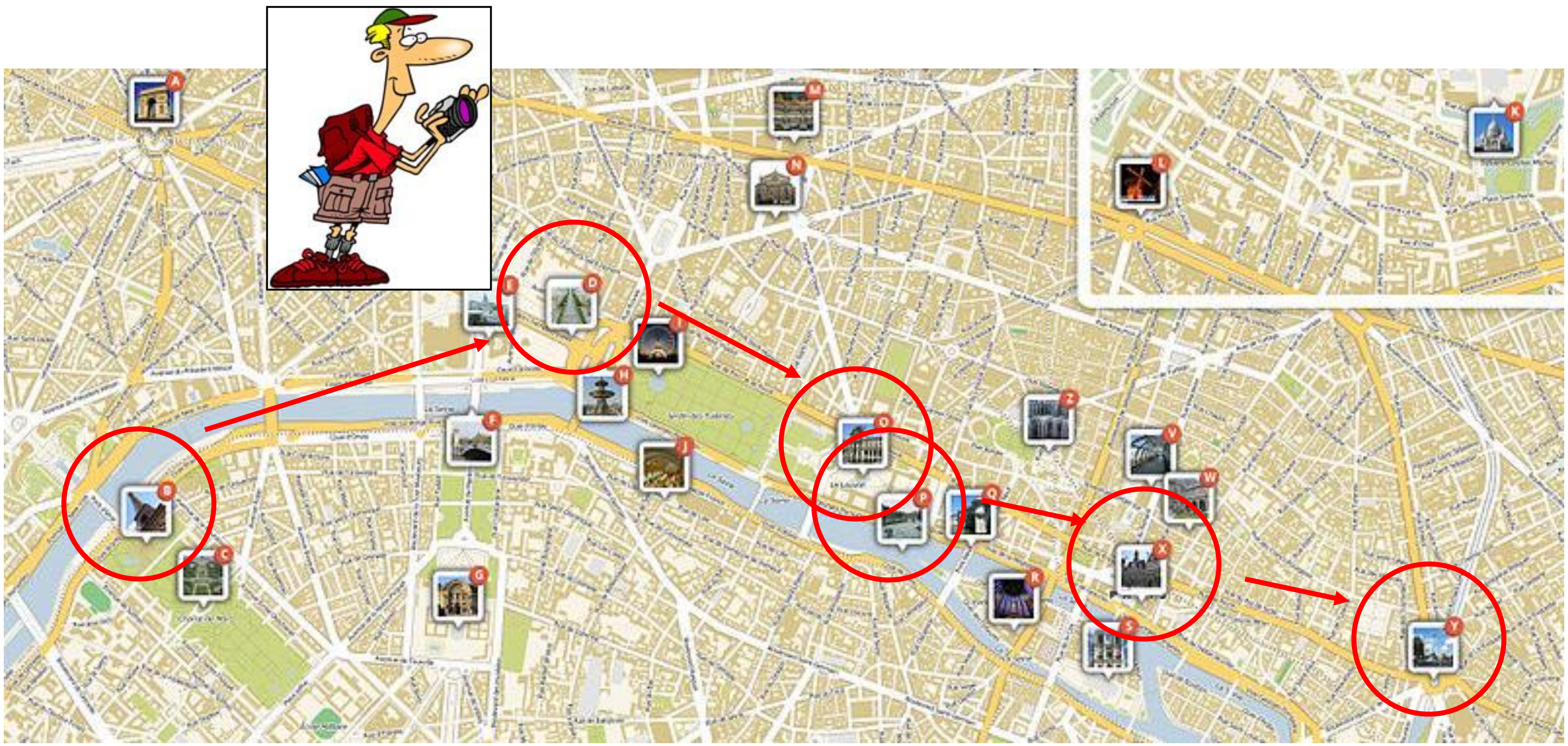
3. Go on the trip
    - travel along the route



Taking a Trip in Real Life

4. Hop off the bus

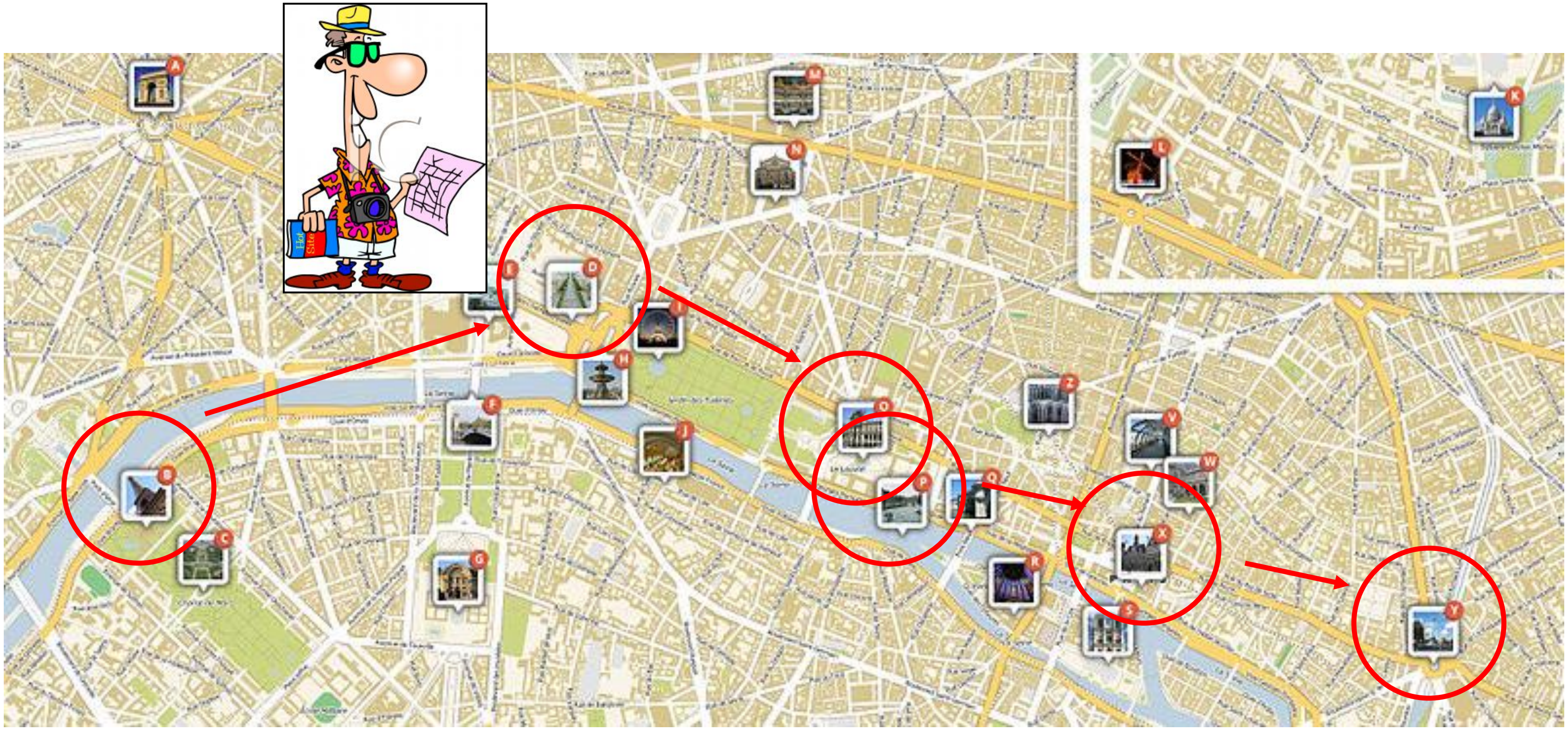
- experience the location, look around, zoom into detail



Taking a Trip in Real Life

5. Orient and localize

- regain bearings in the map



Touring High-D Space…

Exploration goals

- find data configurations that best fit a personal preference in the presence of trade-offs
- find a data partitioning (a clustering) that best fits an exploratory domain model

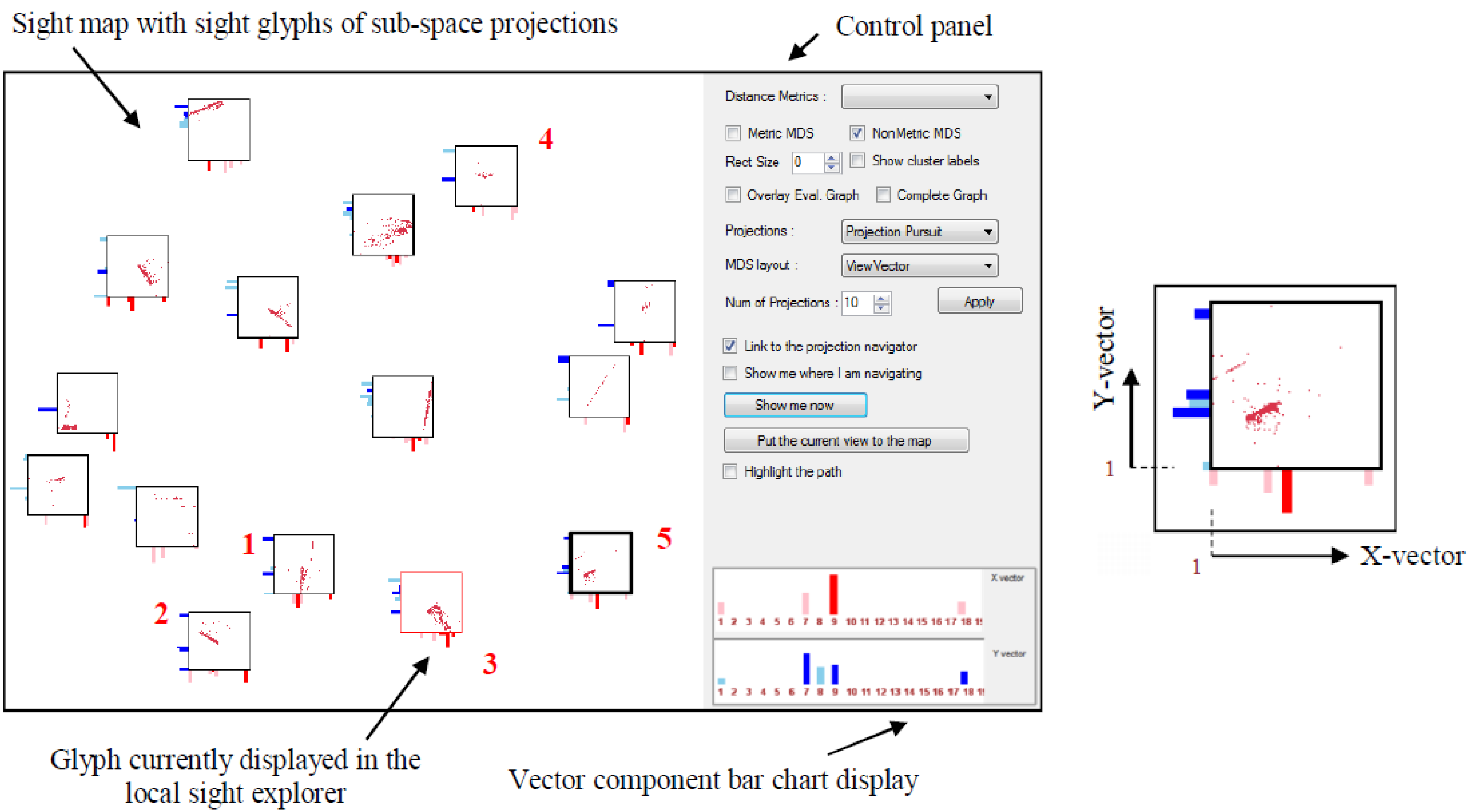
Initial Sights

- key projections as obtained via PCA, projection pursuit, ..
- key clusterings as obtained via k-means, affinity propagation, ..

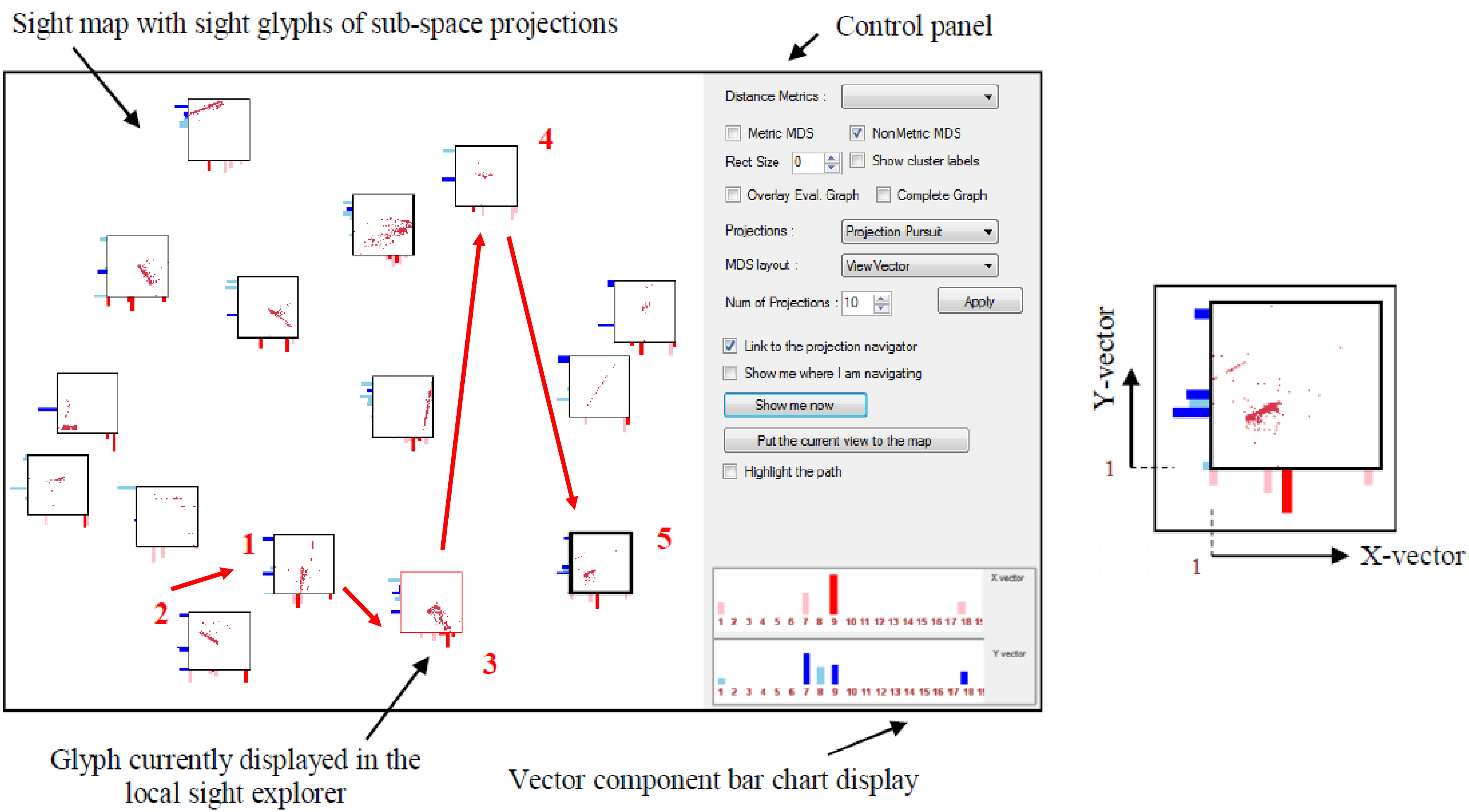
In the tour…

- refine outcome of these automated routines
- tune their parameters to better fit specific goals

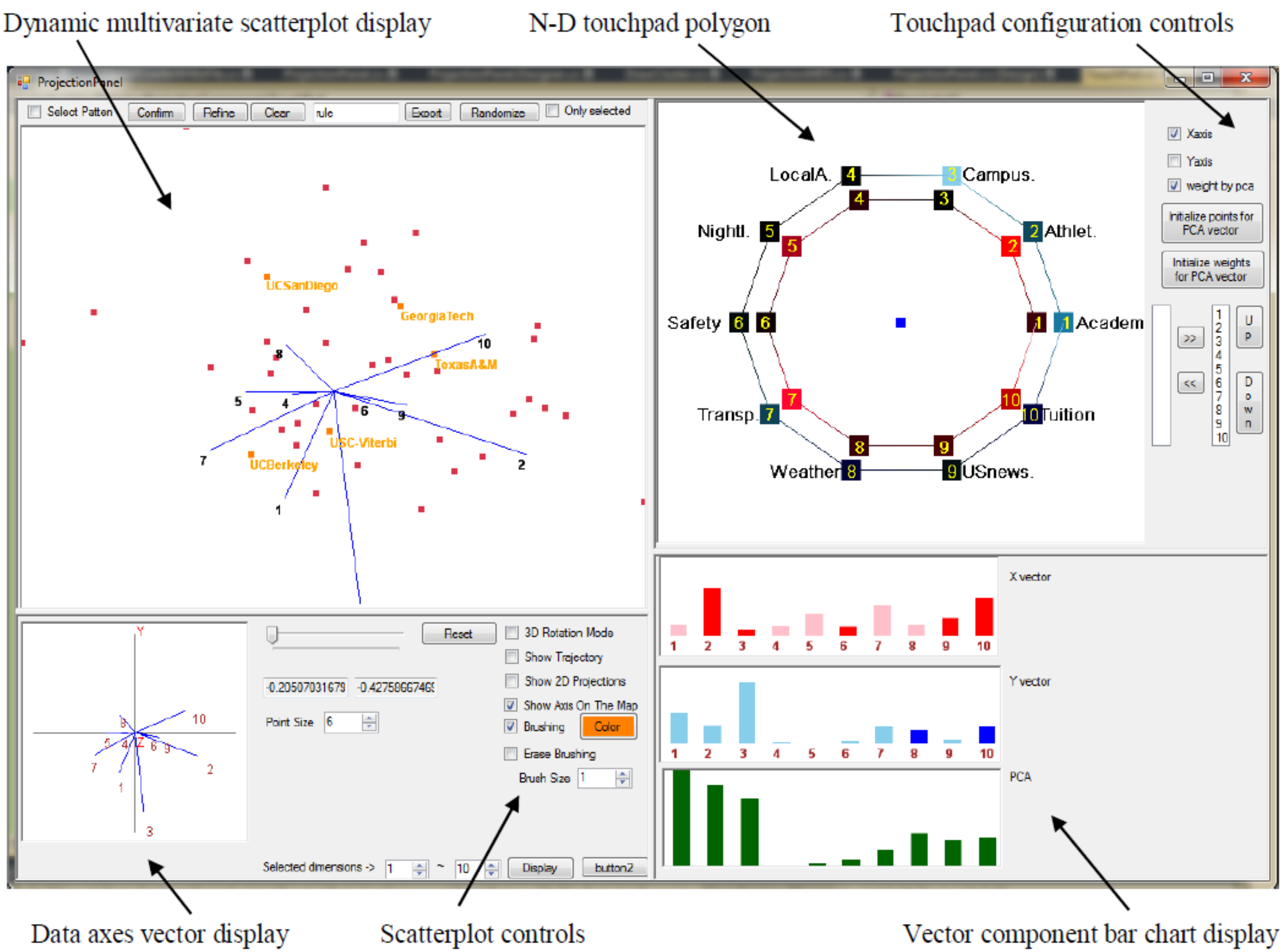
User Interface: Sight Map



User Interface: Sight Map



User Interface: Sight Explorer



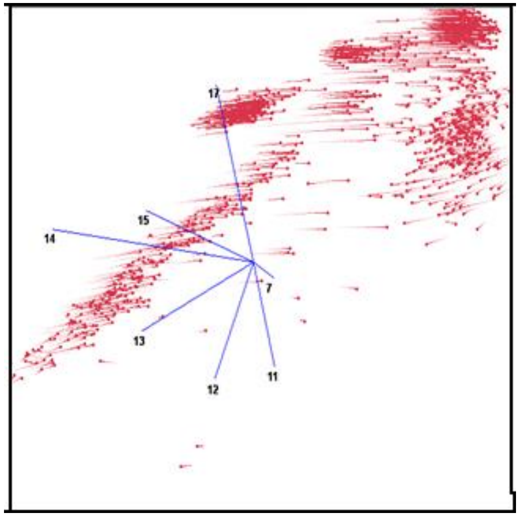
Dynamic Scatterplots

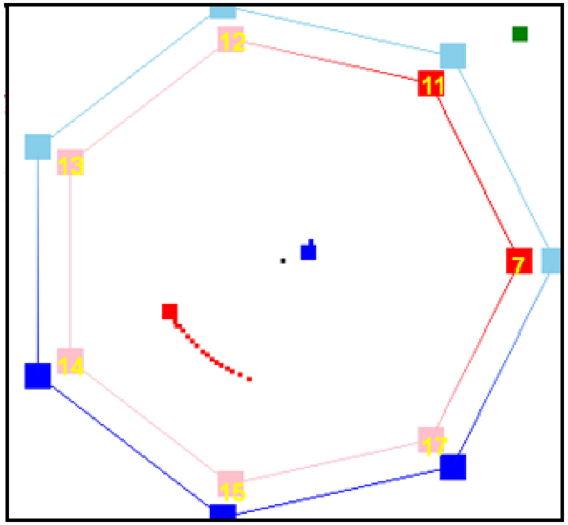
Interaction to help 'see' N-D

- user interface is key → N-D Navigator™

Motion parallax beats stereo for 3D shape perception

- the same is true for N-D shape perception
- help perception by illustrative motion blur





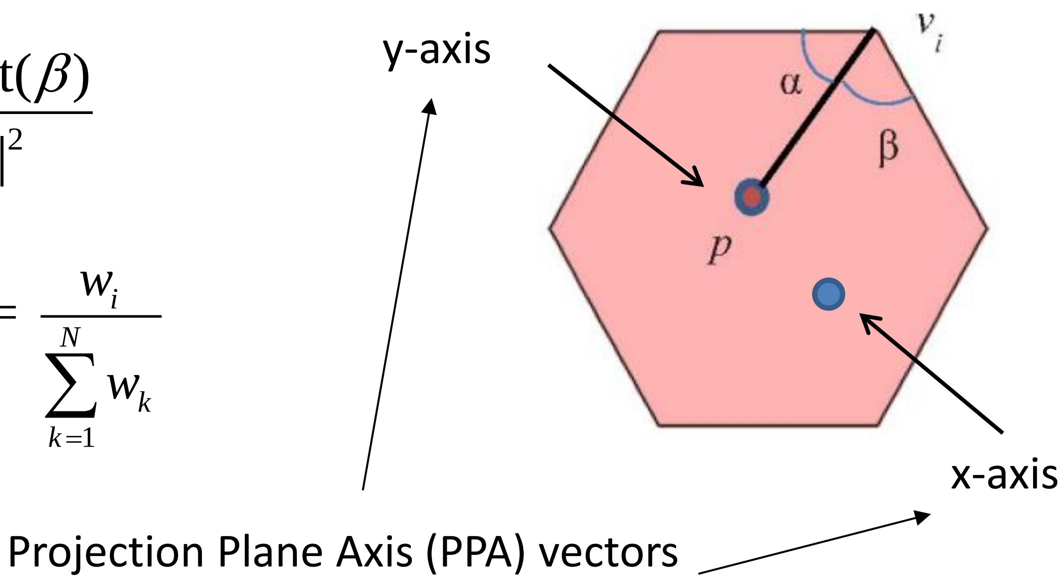
Navigation and Control

Elemental component is the polygonal touchpad

- allows navigation of projection plane in N-D space
- get axis vectors using generalized barycentric interpolation

$$\overrightarrow{w_3} = \frac{\cot(\alpha)+\cot(\beta)}{\| \overrightarrow{p}-\overrightarrow{v_3} \|^2}$$

$$p = \sum_{i=1}^{N} a_i v_i \qquad \text{where } a_i = \frac{w_i}{\sum_{k=1}^{N} w_k}$$



Touchpad Polygon

Vertex order and presence defines the reachable subspace

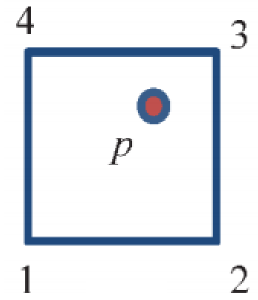
- at least for $M>3$

For a 4D space:

dimensionality	**1**	**2**	**3**	**4**
# hyper spheres	4	6	4	1
hyper sphere dimension sets	1,2,3,4	12,13,14,23,24,34	123,124,134,234	1234, 1342, 1432
touchpad navigation	move to vertex	move along edge	move within triangle	move within poly, re-order

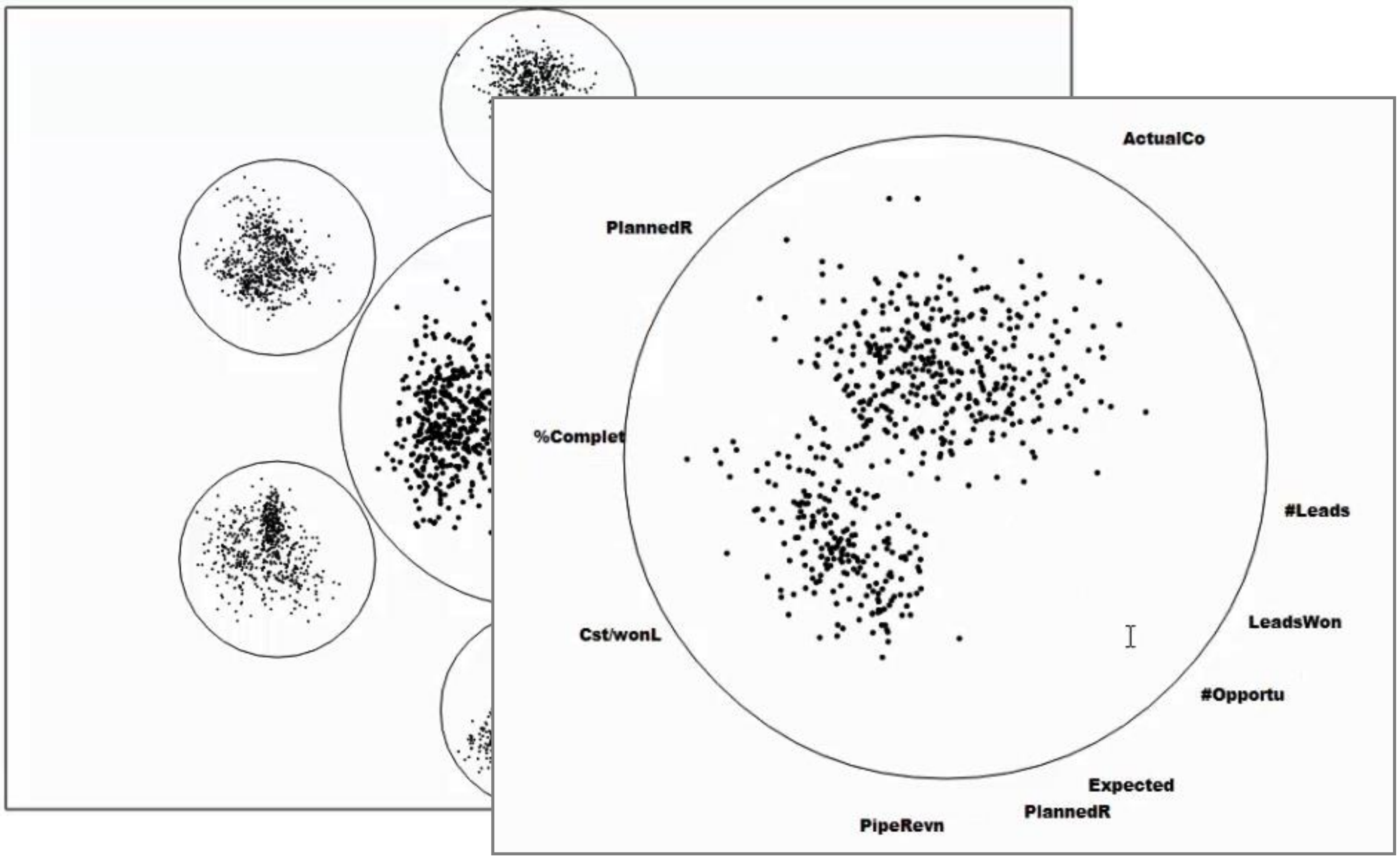
We control this via vertex weights

- setup by prior PCA



Alternative to Touchpad

Directly embed the touchpad into a trackball (under development)



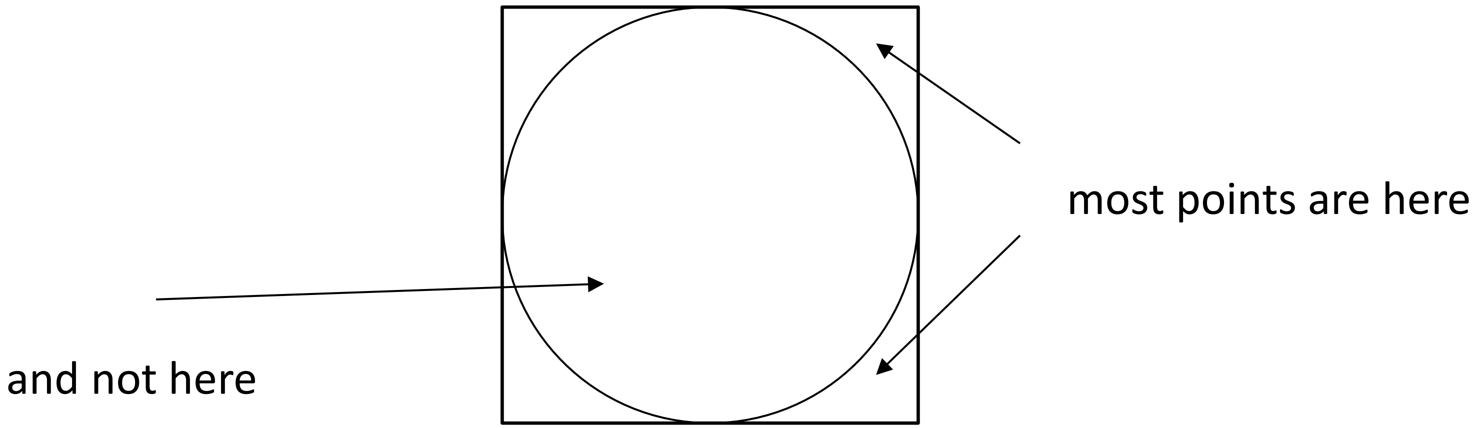
Video

J. Nam, K. Mueller, "TripAdvisor$^{N\text{-}D}$: A Tourism-Inspired High-Dimensional Space Exploration Framework with Overview and Detail," IEEE Trans. on Visualization and Computer Graphics, 19(2): 291-305, 2013. (video)

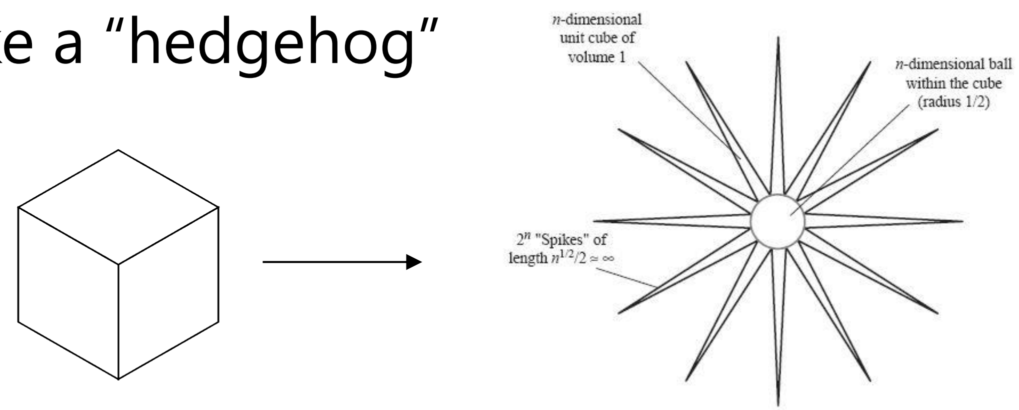
presented at VIS 2012

The Curse of Dimensionality

High-D space tends to be rather sparse



Essentially hypercube is like a “hedgehog”



Relative Contrast

Points are all at about the same distance from one another

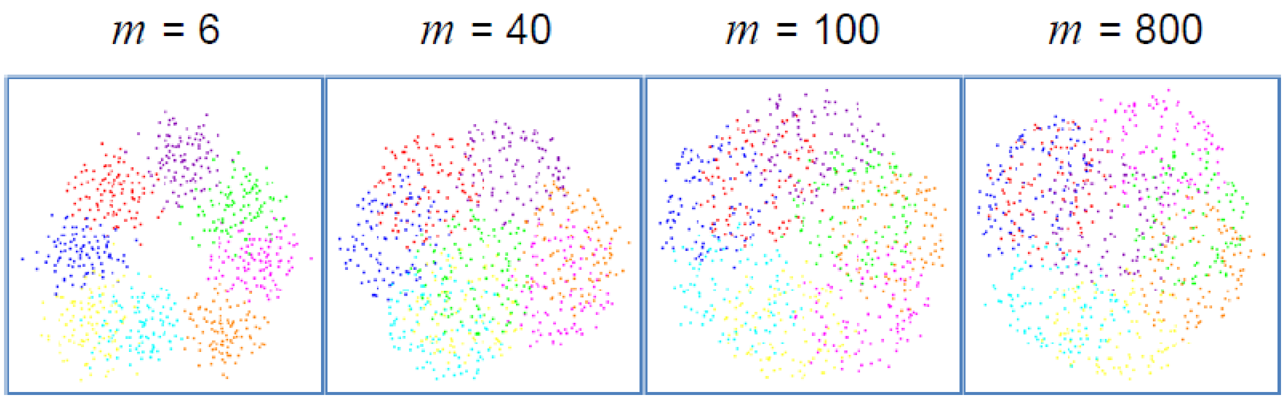
- concentration of distances
- fundamental equation: (Bellman, '61)

$$\lim_{n\to\infty} \frac{Dist_{\max} - Dist_{\min}}{Dist_{\min}} \to 0$$

- so as *n* increases, it is impossible to distinguish two points by (Euclidian) distance
  - points located in the same cluster of points are OK
  - but we need a better distance metric for far-away points

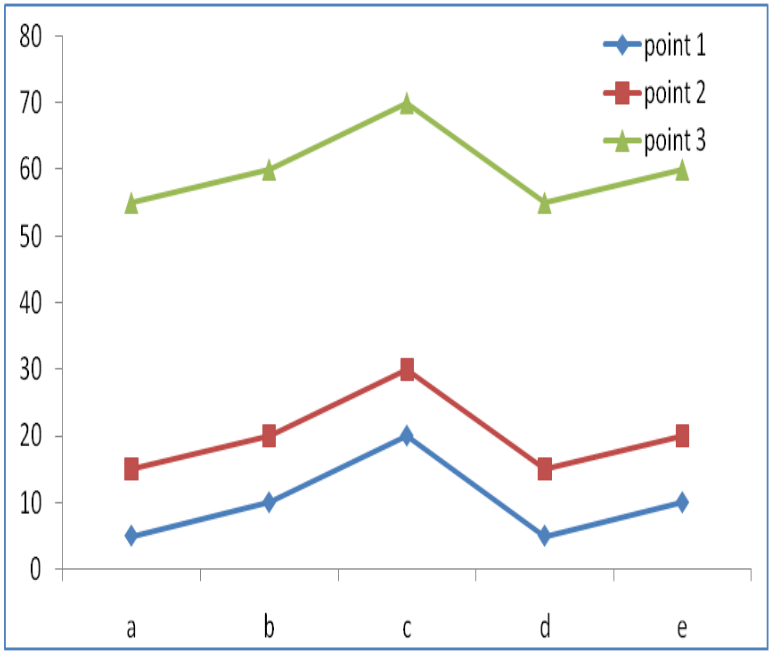
MDS Layout of 8 N-D Gaussian Clusters

Euclidian

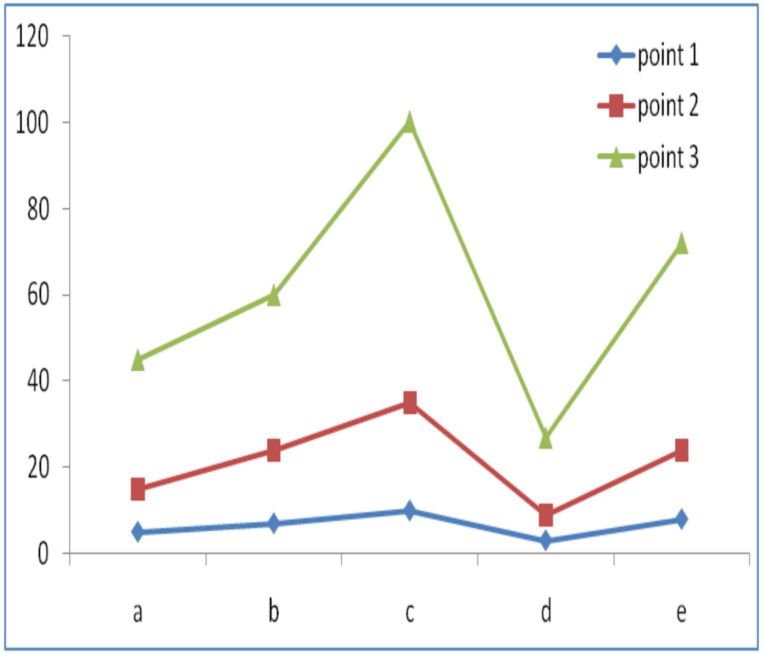


(a) eMDS

Similarity of N-D Points



Same pattern, with offset



Same pattern, with scaling

A Pattern-Based Distance Metric

… with respect to high dimensional signatures

The *Structural Similarity*

$$SSIM = \frac{2\mu_1\mu_2}{\mu_1^2 + \mu_1^2} \cdot \frac{2\sigma_1\sigma_2}{\sigma_1^2 + \sigma_1^2} \cdot \frac{\sigma_{12}}{\sigma_1\sigma_2}$$

similar mean

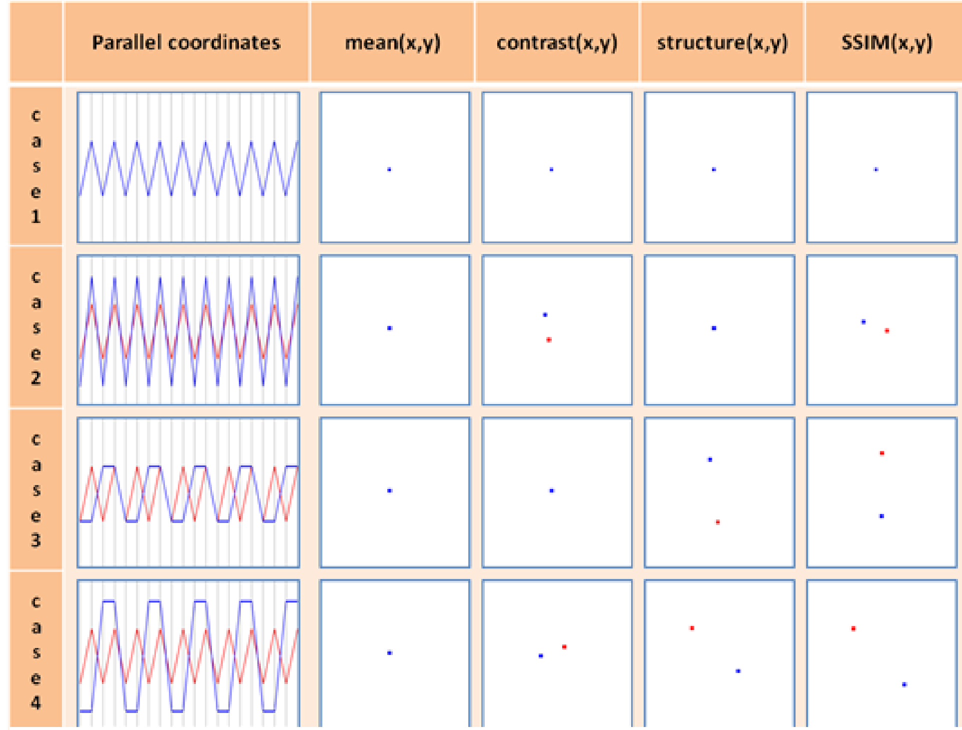
similar variance

good correlation

- maximized when the two points coincide

SSIM Cases

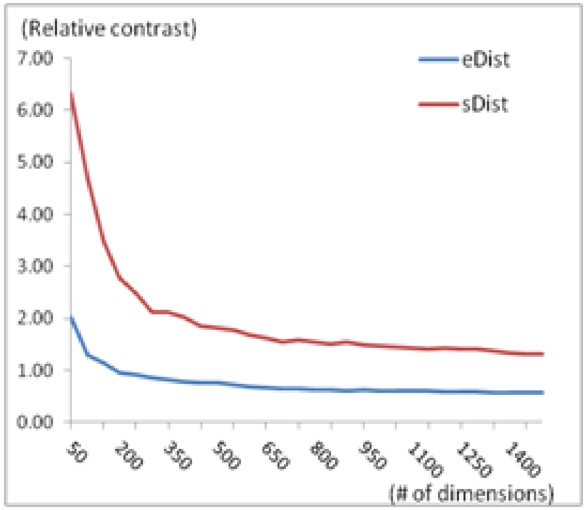
[Same Mean]	Same Variance	Diff Variance
Perfect Correlation	Case1	Case2
Imperfect Correlation	Case3	Case4



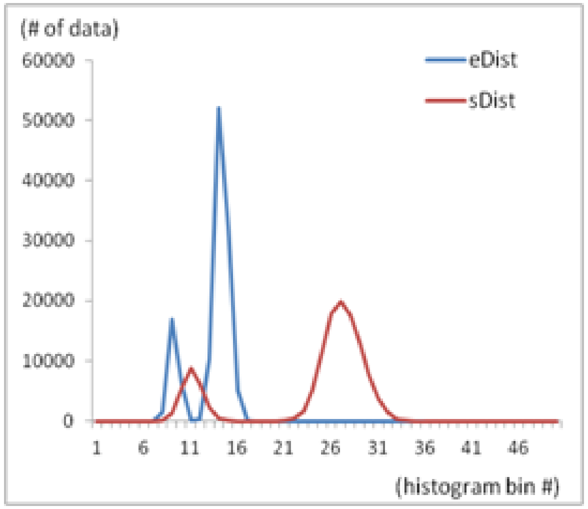
- Perceptual similarity
- Resulting MDS layout good match with parallel coordinate visualization

Improves Relative Contrast

SSIM pushes the limit of the curse of dimensionality

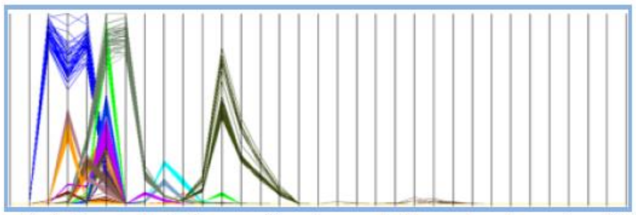


non-clustered

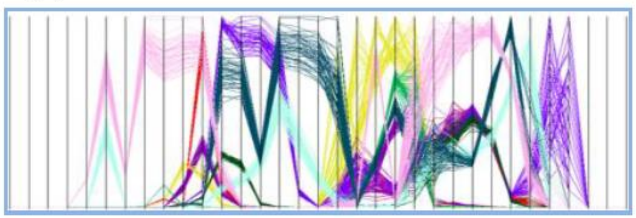


clustered

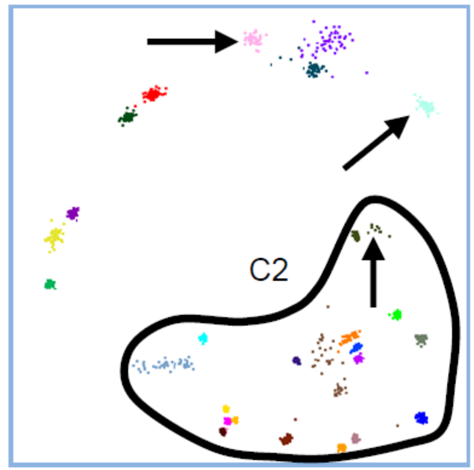
SSIM vs. LDA



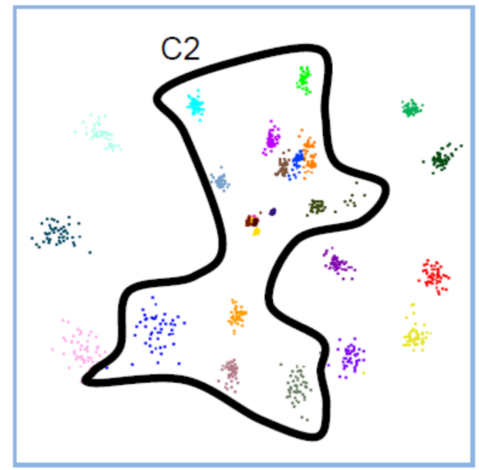
(c) Parallel Coordinates of the clusters in C2



(d) Parallel Coordinates of the clusters not in C2



(a) sMDS

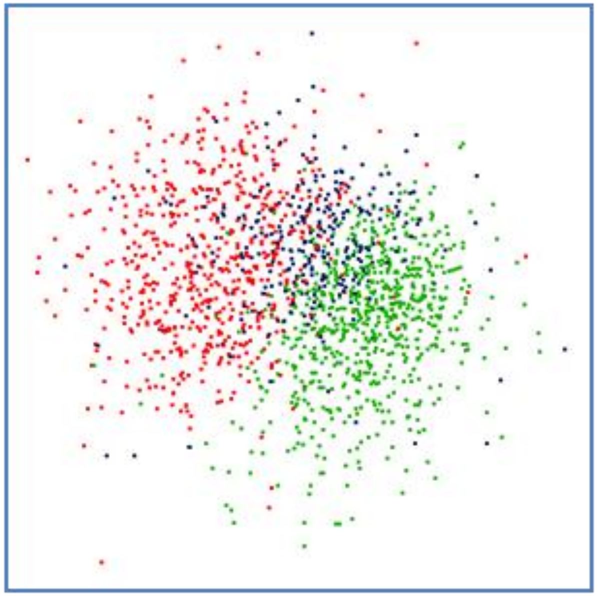


(b) MDS-LDA

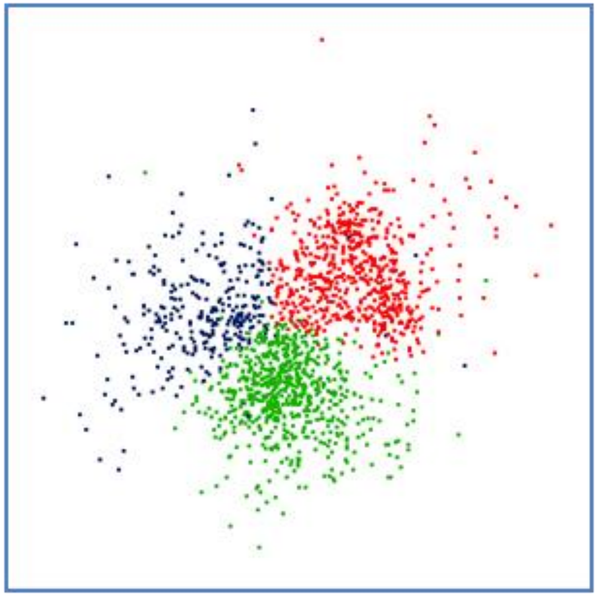
Bi-Scale Layout

Problem with SSIM layout

- good for far-distances (inter-cluster)
- no notion to gauge near-distances (intra-cluster)
- Euclidian distance is more appropriate here



Euclidian



SSIM

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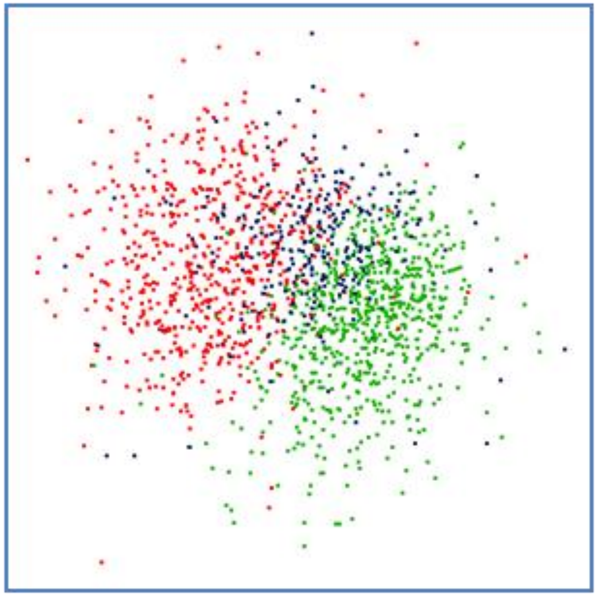
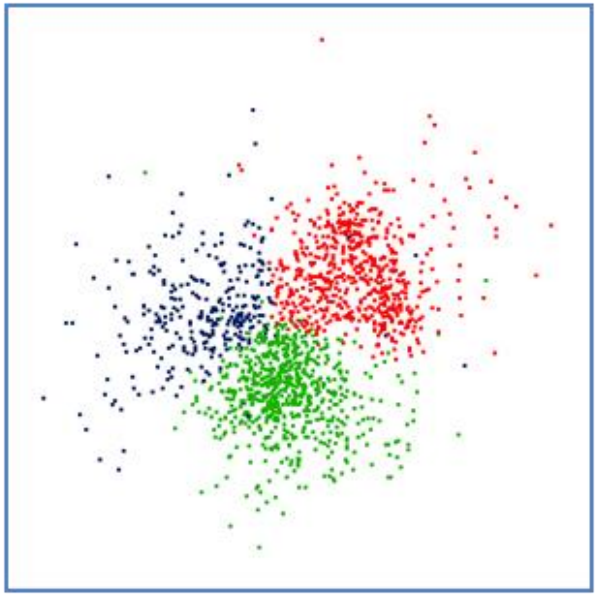
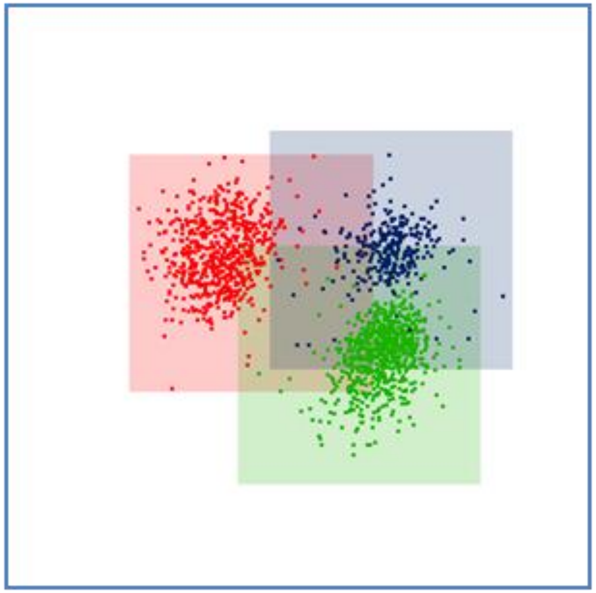


Euclidian

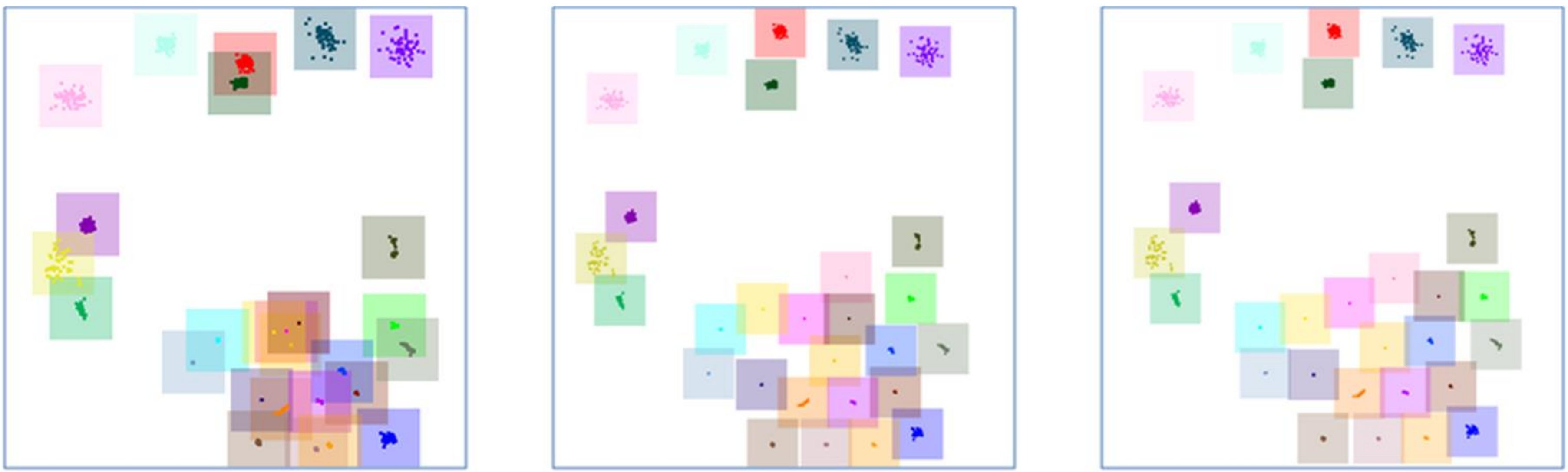


SSIM



Bi-Scale

Bi-Scale Layout Example



(a) No overlap removal (b) $p = 0.2$ (DISSIM = 3.252) (c) $p = 0.05$ (DISSIM = 3.482)

with overlap removal

J. Lee, K. McDonnell, A. Zelenyuk, D. Imre, K. Mueller, "A Structure-Based Distance Metric for High-Dimensional Space Exploration with Multi-Dimensional Scaling," IEEE Trans. on Visualization and Computer Graphics, (to appear), 2014.

N-D Volume Generation

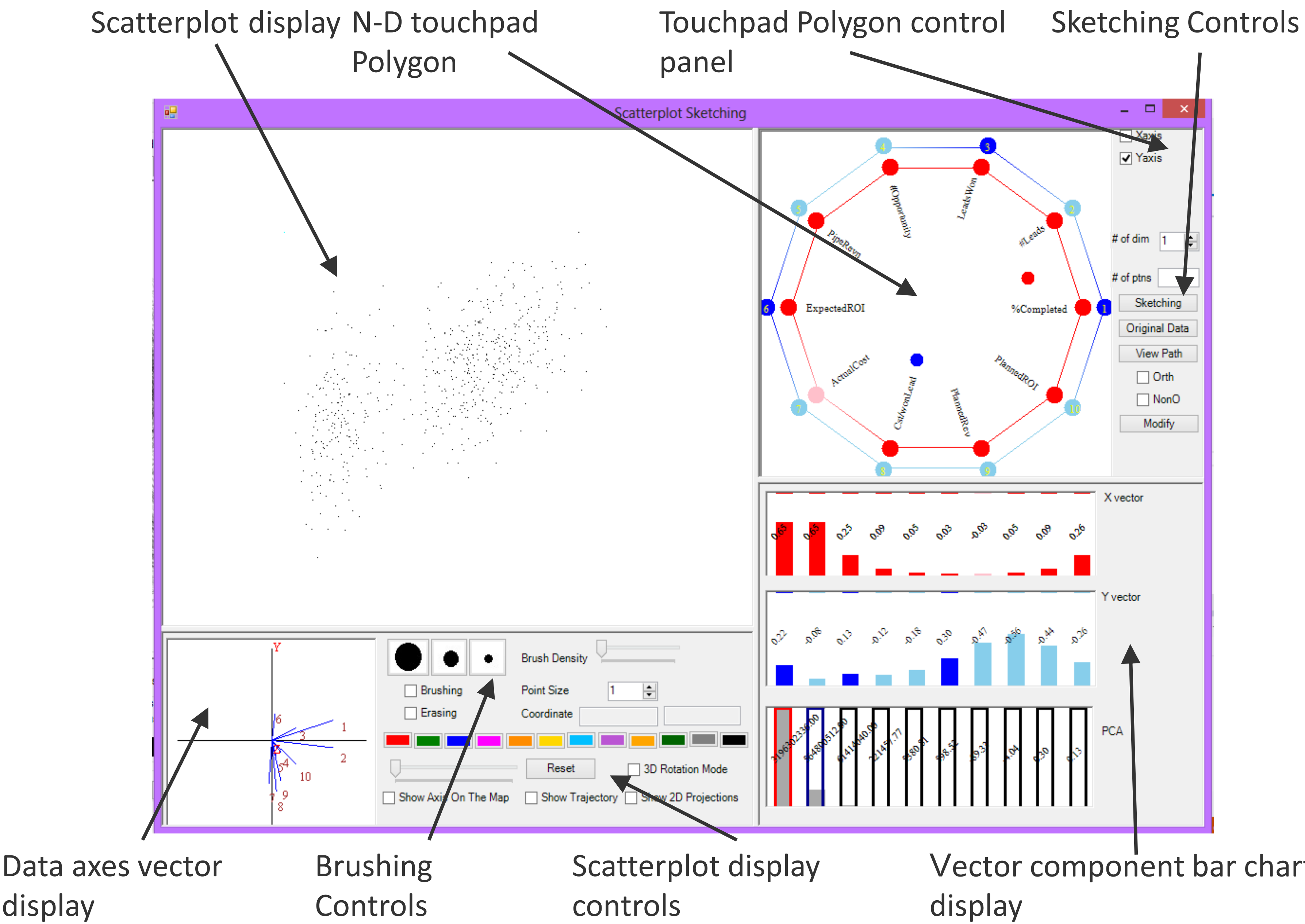
Why Generate N-D Datasets

Testing of new algorithms

Dataset editing and correcting

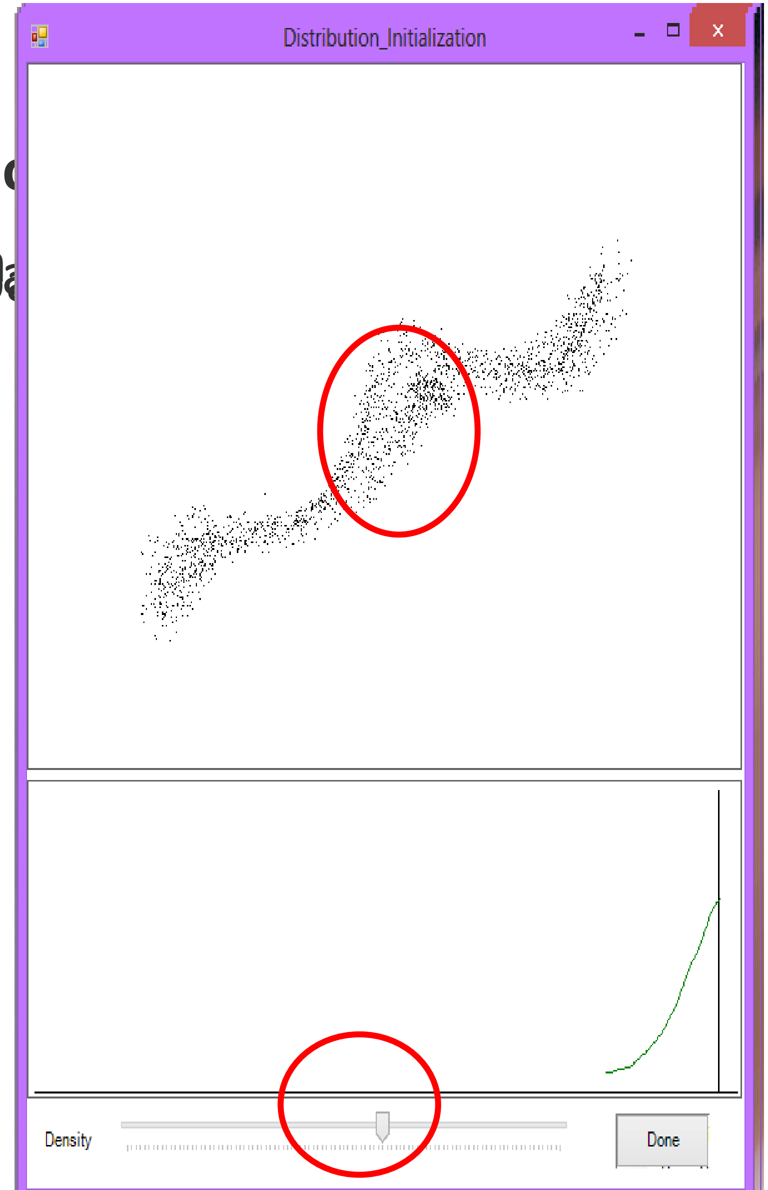
Dataset refinement

Interface


Scatterplot display
N-D touchpad Polygon
Touchpad Polygon control panel
Sketching Controls
Scatterplot Sketching
Xaxis
Yaxis
# of dim
# of ptns
Sketching
Original Data
View Path
Orth
NonO
Modify
#Opportunity
LeadsWon
#Leads
PipeRevn
ExpectedROI
%Completed
ActualCost
CstIwonLead
PlannedRev
PlannedROI
X vector
Y vector
PCA
Brush Density
Brushing
Erasing
Point Size
Coordinate
Reset
3D Rotation Mode
Show Axis On The Map
Show Trajectory
Show 2D Projections
Data axes vector display
Brushing Controls
Scatterplot display controls
Vector component bar chart display

Interaction with Axis-aligned Scatterplots

- **Point Generati**
  - Distributio

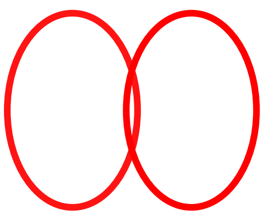


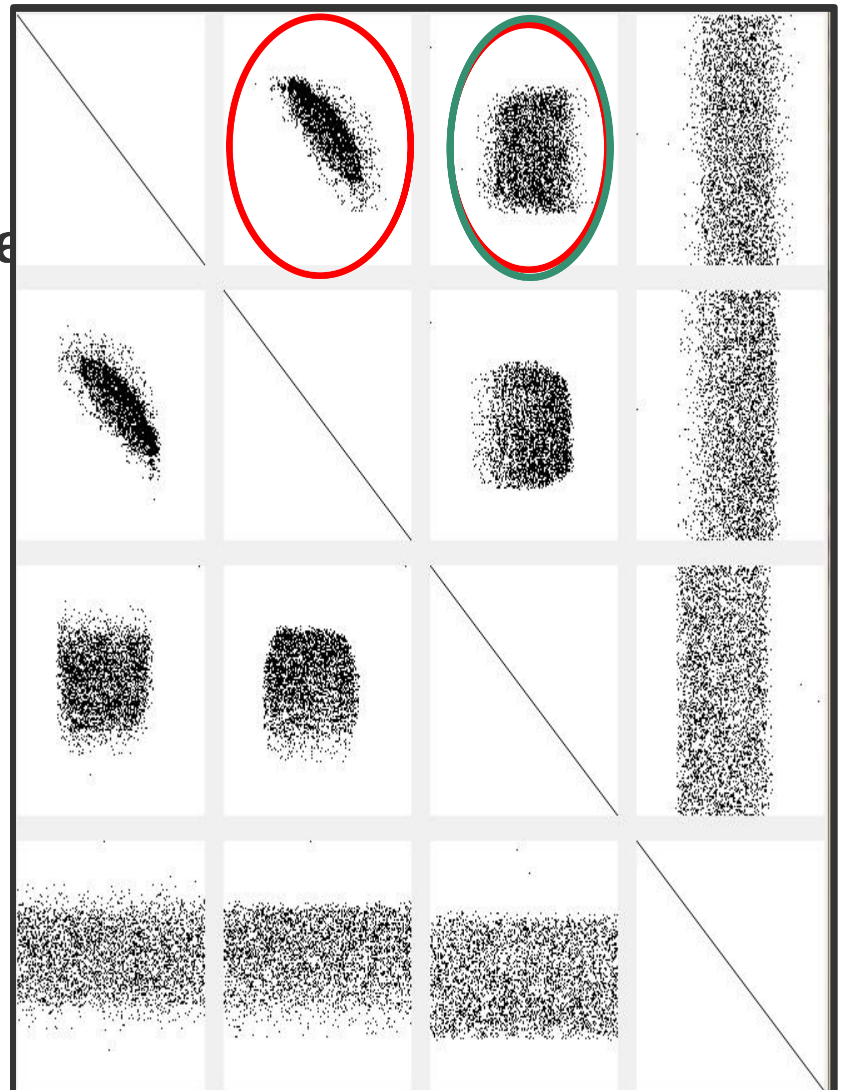
Interaction with Axis-aligned Scatterplots

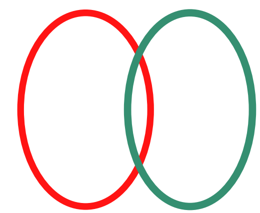
- **Point Generation**
  - Distribution Backprojection
    - 2D distribution → probability map
    - Randomize the values of the other N-2 dimensions

Interaction with Axis-aligned Scatterplots

- **Point Sculpting**
  - Distribution





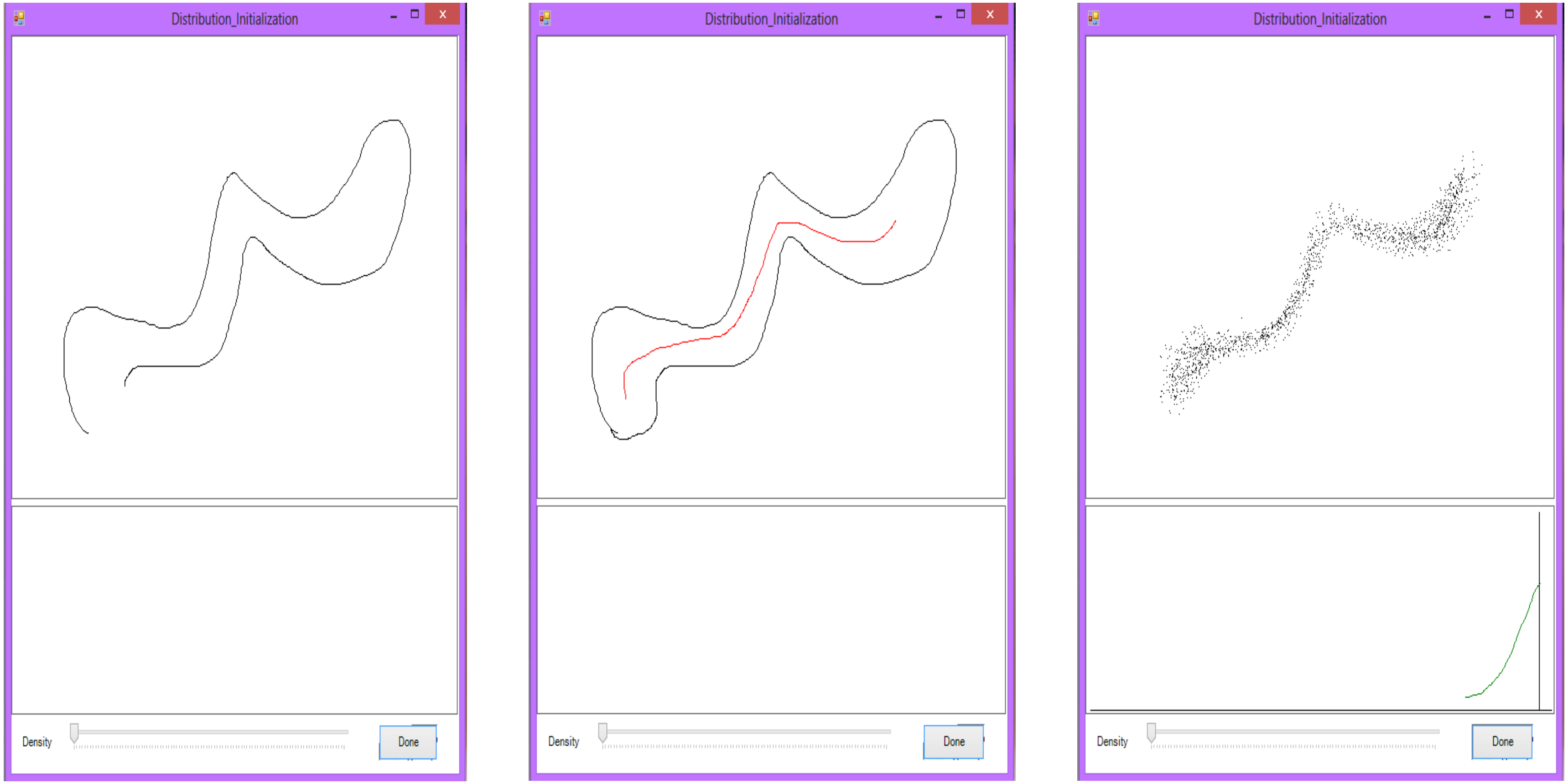


Interaction with Axis-aligned Scatterplots

- **Point Sculpting**
  - Distribution Repair
    - Randomize undefined dimensions’ values
    - Joint probability map of defined dimensions

Interaction with non Axis-aligned Scatterplots

- **Point Generation**



Interaction with non Axis-aligned Scatterplots

- **Point Generation**
  - Distribution Backprojection
    - Gram-Schmidt → new coordinate system

$$y_1 = x_1, \qquad e_1 = y_1/||y_1||$$

$$y_2 = x_2 - proj_{y_1}(x_2), \qquad e_2 = y_2/||y_2||$$

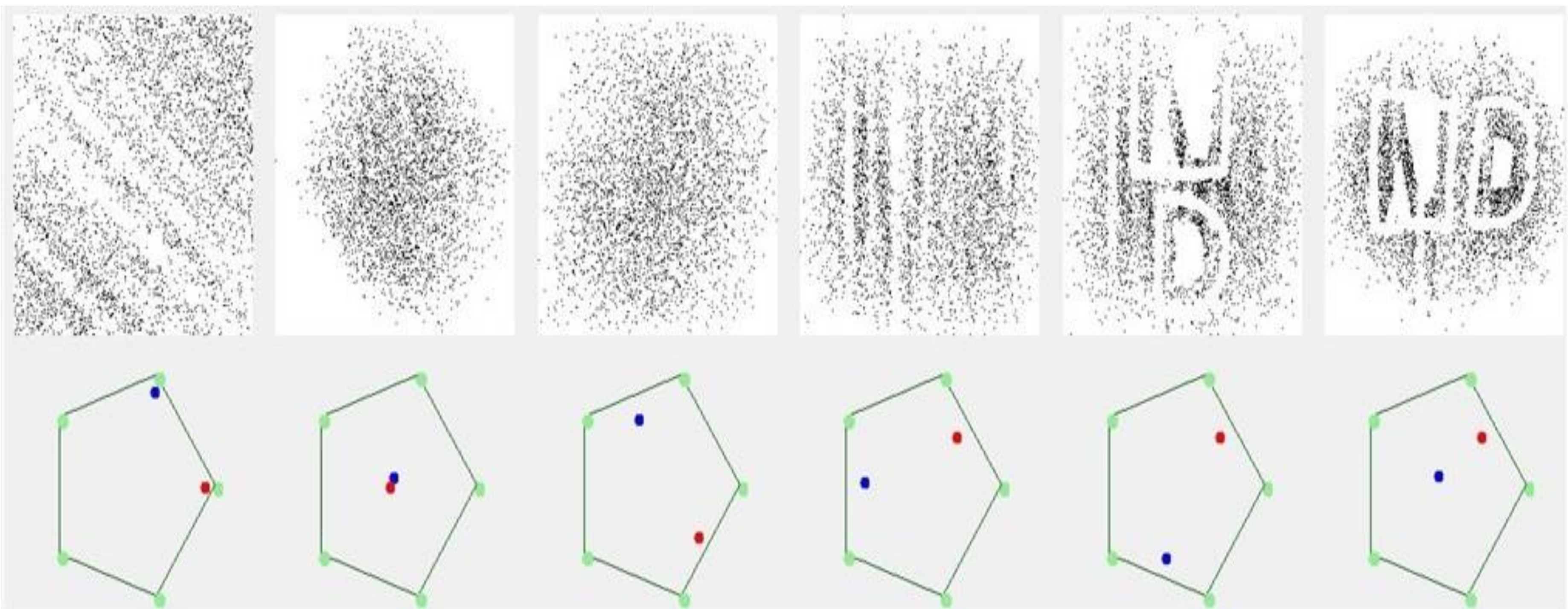
$$y_3 = x_3 - proj_{y_1}(x_3) - proj_{y_2}(x_3), \quad e_3 = y_3/||y_3||$$

$$\vdots$$

$$y_N = x_N - \sum_{j=1}^{N-1} proj_{y_j}(x_N), \qquad e_N = y_N/||y_N||$$

Interaction with non Axis-aligned Scatterplots

- **Point Sculpting**
  - Distribution Carving

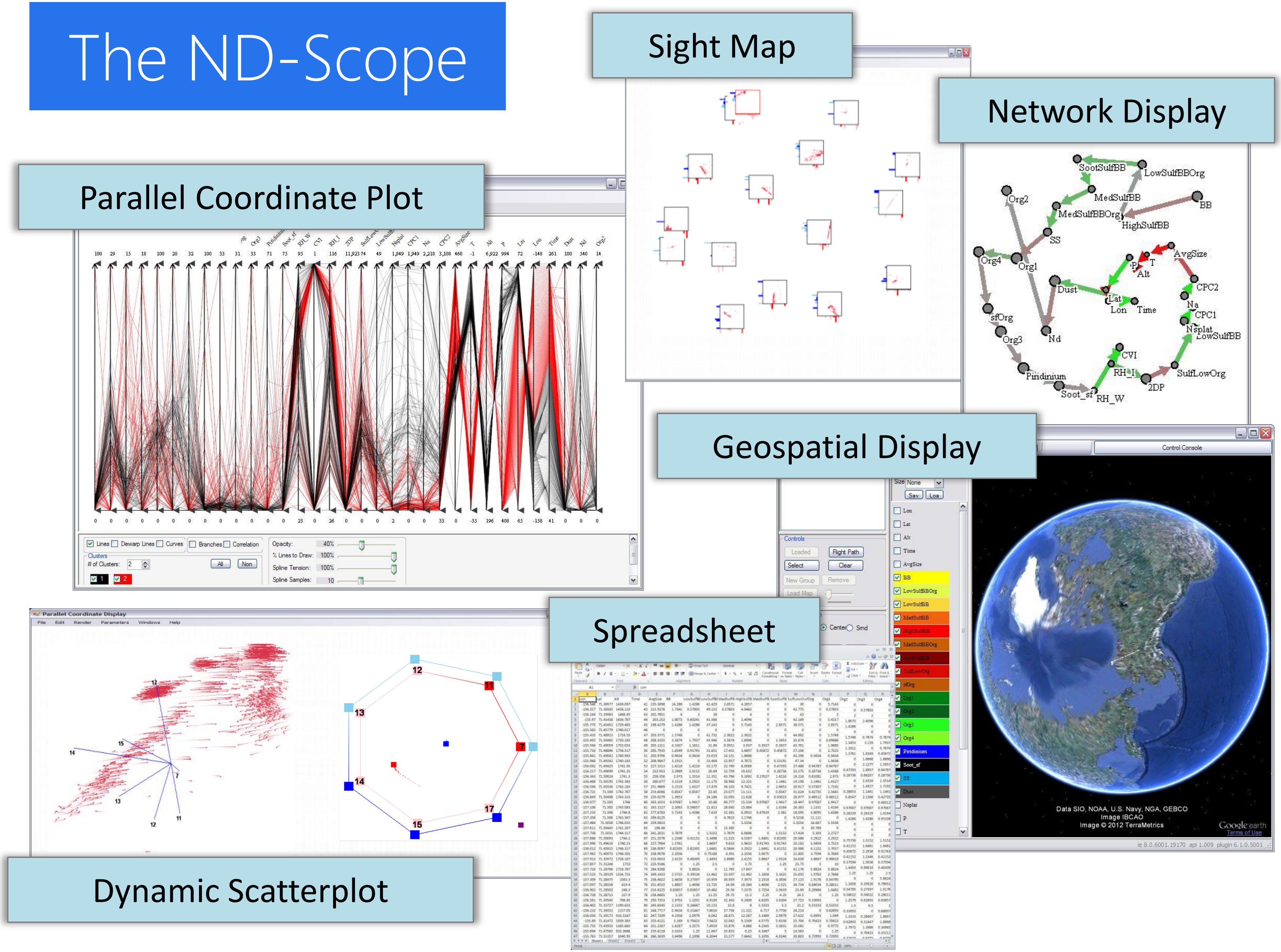


Video

B. Wang, P. Ruchikachorn, K. Mueller, "SketchPad$^{N\text{-}D}$: WYDIWYG Sculpting and Editing in High-Dimensional Space," IEEE Trans. on Visualization and Computer Graphics, 19(12): 2060-2069, 2013. (video)

presented at VAST 2013

The ND-Scope


Sight Map
Network Display
Parallel Coordinate Plot
Geospatial Display
Spreadsheet
Dynamic Scatterplot

Recap

Extending 2D/3D paradigms into N-D has good merit

- makes interactions with N-D data more natural and intuitive
- turns design task into paradigm *extension* and not *invention*
- curse of dimensionality can (most likely, will) cause headache
- but challenges are opportunities and the essence of research

Some examples I've demonstrated

- interaction and navigation via ND touchpad and trackball
- view suggestion and selection via subspace map
- ND distance metric motivated by image perception
- data generation via CT backprojection techniques extended to ND

The good news is…

- there are still many 2D/3D mechanisms and paradigm to port
- rotation matrices, illustrative rendering, shape understanding, …

Support and Collaborators

Support from NSF, NIH, DOE, BNL, PNL, CEWIT, MKE

Faculty:

- Kevin T. McDonnell (Dowling College), IV Ramakrishnan, Erez Zadok, Tamara Berg (SBU Computer Science Department),

Domain scientists:

- Dr. Alla Zelenyuk, Dr. Dan Imre (PNL), Yangan Liu (BNL)

PhD students (at Stony Brook and SUNY Korea):

- Zhiyuan Zhang, Hyunjung Lee, Nafees Ahmed, Bing Wang, Puripant (Joe) Ruchikachorn, Sungsoo Ha,, Jisung Kim, Eric Papenhausen, Salman Mahmood, Shenghui Cheng, Ziyi Zheng (PhD, now at Amazon). Julia EunJu Nam (PhD, now at Microsoft), Wei Xu (PhD, now at BNL)

Relevant References

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JB. Wang, P. Ruchikachorn, K. Mueller, "SketchPadN-D: WYDIWYG Sculpting and Editing in High-Dimensional Space," *IEEE Trans. Vis. and Computer Graphics*, 19(12): 2060-2069, 2013.

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http://www.cs.stonybrook.edu/~mueller

(for videos see dedicated paper web pages)

http://nd-scope.net

Any questions?